A COMPUTER IMPLEMENTATION OF THE MUNSELL COLOR SYSTEM

By

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PREFACE

This thesis is the description of the methods employed in a computer program which implements the Munsell Color System. The original objective for the implementation was to aid the Oklahoma State University Meat Laboratory in its research to determine the effects of temperature on the color of meat. It is hoped that the program will be utilized by other departments whose research touches on the area of color designation.

The author wishes to express his deep appreciation to his mother and father without whose parental guidance any of this advanced education would not have been possible -- especially to his father whose backing and advice were always available whenever needed.

I would like to thank my major advisor, Dr. D.W. Grace for his probing questions and useful suggestions concerning the content of this thesis.

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TABLE OF CONTENTS

Chapter		Page
I.	INTRODUCTION	1
	Description of Thesis	1
II.	AN OVERVIEW OF COLOR	3
III.	THE RGB AND XYZ COLOR SYSTEMS	8
IV.	THE MUNSELL COLOR SYSTEM	31
V.	PROGRAM DEVELOPMENT	56
VI.	RESULTS AND CONCLUSIONS	82
A SELECTED BIBLIOGRAPHY		87
APPENDIX		89
	USER'S GUIDE	89

LIST OF TABLES

Table		Page
I.	Definition of Color in Terms of Wavelength	5
II.	Munsell Value Steps V and Relative Luminance Factors Y_r With Respect to Magnesium Oxide M_gO	40
III.	I.C.I. (Y) Equivalents (in Percent Relative to M_gO) of the Recommended Munsell Value Scale (V) From 0/ to 10/	42
IV.	Spectrum Locus Data	63
V.	List of Pc (5/5) Values	81
VI.	List of Results Produced by the Computer Program	85

LIST OF FIGURES

Figure		Page
1.	Breakdown of Electromagnetic Radiations	3
2.	Spectrum of Visible Light	4
3.	Diagram of Hue, Saturation and Lightness	6
4.	Color Circle of Sequential Color Chips	10
5.	Diagram of Mixture of Red and Yellow Color	11
6.	Young's Color Triangle	11
7.	Light Response Curves	12
8.	Color Matching Diagram For White Light	14
9.	Color Matching Diagram For Orange Light	15
10.	Curves For the Equal Energy Spectrum	18
11.	Arrangement For Using Spectrum Lamps to Duplicate the Spectral Energy Distribution of a Source and a Transparent Object	19
12.	Diagram of Relative Energy Reaching the Eye of a Normal Observer	21
13.	Energies of Spectrum Lights Adjusted to Match the Object's Spectral Energy Distribution	22
14.	How to Obtain C.I.E. Tristimulus Values	24
15.	C.I.E. Color-Matching Functions	26
16.	Chromaticity Diagrams of Color Triangles Derived From the C.I.E. 1931 XYZ System	27
17.	Spectrum Locus and Standard Illuminants	29
18.	Color Print of Spectrum Locus	30
19.	Munsell Hue Color Notation	33

Figure Page

20. Munsell Value Axis 34

21. Diagram of Munsell Chroma 35

22. Munsell's Three Dimensional Concept of Color 36

23. Diagram to Illustrate Munsell Hue, Value and Chroma in Their Relation to One Another 37

24. Diagram of x,y Chromaticity and Luminance Y Value 39

25. How the Munsell Value Function Varies With Luminance Y . . . 41

26. Hue and Chroma Loci, Munsell Value 1 43

27. Hue and Chroma Loci, Munsell Value 2 44

28. Hue and Chroma Loci, Munsell Value 3 45

29. Hue and Chroma Loci, Munsell Value 4 46

30. Hue and Chroma Loci, Munsell Value 5 47

31. Hue and Chroma Loci, Munsell Value 6 48

32. Hue and Chroma Loci, Munsell Value 7 49

33. Hue and Chroma Loci, Munsell Value 8 50

34. Hue and Chroma Loci, Munsell Value 9 51

35. Illustration of Methods of Determining Dominant Wavelengths λ_d and Complementary Wavelengths λ_c 52

36. Master Hue Chart in I.C.I.(x,y) Coordinates 55

37. The Principle of the Photoelectric Tricolorimeter 57

38. Basic Flowchart for Munsell Color Program Development . . . 59

39. Straight Line Intersecting Part of Spectrum Locus Data Points 65

40. Intersection of Two Straight Lines 66

41. Intersection of a Straight Line and a Quadratic 66

42. Intersection of a Straight Line and a Quadratic Which is the Average of Two Quadratics 67

43. Rotation of Quadratic Points 67

Figure Page

44. Straight Line Fit to a Given Four Points 68

45. Unique Quadratic Passing Through Three Hue Points 75

46. Angles Calculated By ATAN2 Function 76

47. Method of Bracketing Sample Point With Hue Lines 77

48. Case Where Sample Point Should Move Down One Hue 78

49. Case Where Sample Point Should Move Up One Hue 78

50. Computer Output Sample 92

51. Sample Data Card 93

52. Trailer Card For Data Deck 93

53. Sequence of Cards to Run the Color Program 94

CHAPTER I

INTRODUCTION

Description of Thesis

Data from a colorimeter can be transformed into a color representation, called the Munsell color notation, by means of a simple transformation. The transformation involves the use of specified tables and graphs, often accompanied by some form of interpolation. The results obtained give an accurate determination of a given object's color. This thesis will describe the implementation of this transformation.

The Oklahoma State University Meat Laboratory runs experiments to determine what the effect of temperature is on the color of meat. The meat is analyzed by a three filter colorimeter. The data recorded is then converted into a color representation known as the Munsell Color system. This system requires the use of tables, graphs and at times some form of interpolation, thereby introducing the element of human error.

This thesis describes a procedure for converting the data produced by the colorimeter into the corresponding Munsell color notation. Chapters II and III give a brief overview of light and other electromagnetic radiations. Chapter IV describes the Munsell system and its use. The Munsell notation is easy to understand and convenient for laboratory work. Chapter V describes the program development, the problems encountered in the development and the solutions used to remove

these problems. Chapter VI lists the results and conclusions drawn from the programs results.

This thesis is an extension of the report DATA CONVERSIONS IN COLORIMETRY by Domingo Antonio Russo. Russo's program was confined to the red part of the visible spectrum. It was felt that this was a restriction and that the program could be expanded to cover the whole of the visible spectrum and in order to facilitate applications in areas other than meat grading. It is hoped that the program could then be used by other departments, who deal with color recognition in other parts of the spectrum. The Forestry department could use it when comparing leaf colors. The Agriculture department could use it when grading wheat or any type of farm produce. The Chemistry department could use it for experiments whose results are based on color. In fact, any department whose research uses color comparisons could utilize the program.

It should be realized that there is still a great deal of work to be done in the area of color definitions. The data which was used to generate the tables and graphs used in the Munsell color system date back to the 1930's. These same tables and graphs are used in color calculations today. Hopefully in the future, with the results of new research these graphs, tables and color notation methods will be updated. The program was developed so that new tables can replace the old ones within the program with the minimum amount of effort.

Appendix A contains a user's manual to aid in the use and the running of the computer program.

CHAPTER II

AN OVERVIEW OF COLOR

To understand color and its implications the fundamental aspects of light must be understood. Light is a type of electromagnetic radiation. Electromagnetic radiation is known to us under many names. Figure 1 illustrates the composition of the spectrum of electromagnetic radiation.

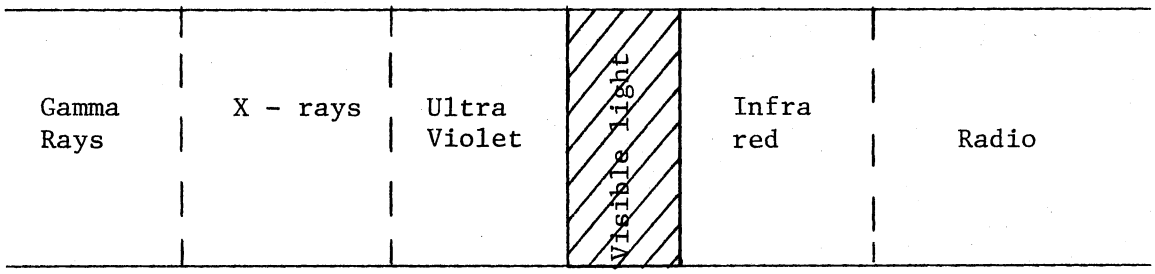


Figure 1. Breakdown of electromagnetic radiations (5).

The human eye can only respond to the wavelengths contained in the hatched part of the diagram. It is within these wavelengths that color as we know it, is perceived.

Color is determined by the intensity of the various frequencies in the visible spectrum. A color perceived by the human eye is a combination of these radiations with perhaps one radiation dominant over the others. Figure 2 illustrates this.

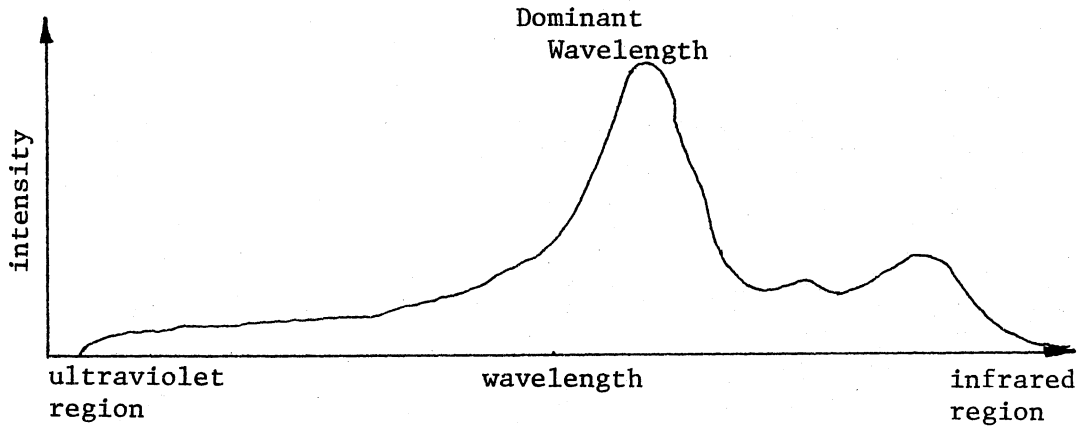


Figure 2. Spectrum of visible light.

The three basic monochromatic wavelengths which affect the eye are those of red, green, and blue. To the eye the monochromatic radiations are distinguished by the color sensations they evoke. They display the well-known spectral colors that can be observed in the rainbow: red, orange, yellow, green, blue, indigo, violet.

Table I gives a list of monochromatic wavelengths and the divisions into which they fall. Electromagnetic radiations with wavelengths smaller than about 380 nm or larger than about 780 nm are, in general, invisible to the human eye.

Color may be described in terms of three variables called hue, saturation, and lightness.

Hue may be described as the main quality factor in color. It is this factor in color which leads us to name an object red or green. As the combination of the visible spectral wavelengths changes so does the hue. Hue is the chief, but not the only, characteristic that gives rise to the basic color names.

Saturation may best be defined as the percentage of hue in the color. In fact saturation is a measure of the realtive amount of white

TABLE I

DEFINITION OF COLOR IN TERMS OF WAVELENGTH

Wavelength (nm)	Color name
380	
	violet
436	
	blue
495	
	green
566	
	yellow
589	
	orange
627	
	red

(Unit nm 1 nanometer 1 millionth of a millimeter.) These definitions of color in terms of wavelength are arbitrary, and are not universally agreed upon.

light in the color. The greater the percentage of white light contained in a hue, the less saturated the hue is said to be.

Lightness describes the appearance of an object in terms of its apparent amount. Figure 3 gives a schematic diagram of how to visualize hue, saturation and lightness.

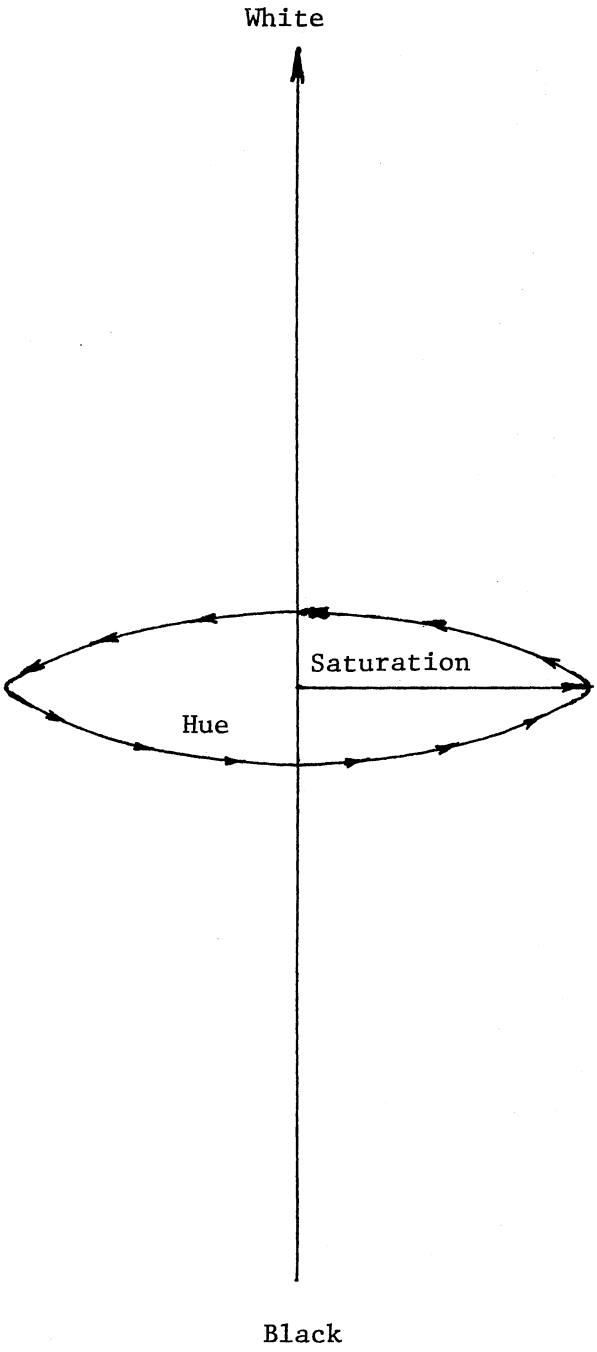


Figure 3. Diagram of hue, saturation and lightness.

"The central vertical axis represents the locus of greys, with black at the lower and white at the upper extremity. The distance from black to white fixes the scale of the solid. The height of a sample above the black level will be determined by its lightness, while its distance from the black-white axis indicates its saturation. If the plane is assumed to rotate around the black-white axis, it will pass through successive hues of red, orange, yellow, etc." (10)

The orderly description and specification of color is an essential part of solving color problems. A universally accepted system is needed to express our ideas about the appearance of color. There are in existence today many systems which try to do this. Chapter II will briefly describe these systems, leading up to the Munsell system which is implemented in the computer package.

From Figure 2 we see that color is really determined by the entire curve giving intensity as a function of wavelength. This curve is composed of an infinite number of points; in fact a function is often considered to be a vector in a Hilbert space having an infinite number of dimensions. The various color systems such as the Munsell system evidently approximate these intensity functions by a vector in a space of only three dimensions. This approximation can never be perfect. That it succeeds at all probably is due mainly to the physiological limitations of the human eye.

Note also that in Figure 3 a circle near the bottom of the diagram will encompass all hues, but all colors on the circle are nearly black and hence are nearly indistinguishable by the eye. Kender (14) has characterized this situation as an essential singularity in the system. There is no known way to remove the singularity, although that would seem to be desirable.

CHAPTER III

THE RGB AND XYZ COLOR SYSTEMS

The principles of color were developed in the previous chapters. Color descriptions can vary from person to person, making color a very uncertain subject, unless qualified in some way. To overcome this problem, suppose a set of standard tables of distribution coefficients were to be adopted. These tables are to represent the color-matching characteristics of the average eye. Once a set of coefficients is agreed upon, then color specifications can be derived which are independent of the color vision of any particular individual. Before such a system of color measurement can be established, though, agreement on two separate issues is required (11). These are:

1) that the preparation of a standard system of color measurement should be based on the color-mixture data representative of a normal observer

and

2) there should be an agreement on a set of reference stimuli relative to which the distribution coefficients can be transformed and in terms of which standard color specification can be expressed.

In 1931 the CIE (Commission Internationale de l'Eclairage) considering the above constraint recommended a new system of color co-ordinates. The system is so arranged as to make practical calculations possible in a simple way. The system is based on the three reference stimuli, red 700 nm, green 546, nm, and blue 436 nm, and is known as the RGB system. From the RGB system, a second, more usable

system, the XYZ system, is derived. Before an explanation of the RGB, XYZ systems is given, the reader should first of all be acquainted with some background material.

The visible part of the energy spectrum is based on monochromatic colors. These monochromatic colors make everything that can be seen visible. Every color stimulus imaginable is the result of a mixture of these colors. Their interaction is responsible for the wide variety of the manifestations of color. The spectrum of colors the human eye perceives is a mixture of the three basic primaries -- red, green, and blue. How then do we obtain the mixture of primary colors contained in that color?

Suppose that you were confronted with a large number of colored chips of red, green, blue, pink, orange etc. and were asked to order them in terms of their similarity. In the process of creating such an ordering of colors, you will decide that orange is more similar to red than is yellow, and accordingly you place the reds and oranges next to each other. Similarly, the yellows resemble the oranges more than they do the greens and blues, and thus the yellow would be placed near the oranges. This sequencing goes on until you have ordered all of the chips. If you are very clever you will realize that the entire array of chips can be arranged in a circle (6). How is it possible that a mixture of two of these spectral colors can produce the impression of a third spectral color?

Suppose that we mix two colors on the circumference of the circle in certain proportions. For example let us mix the colors red and yellow. If we include more yellow than red, the resulting color will be "yellower" than if the two colors are mixed in equal amounts. If we include more red than yellow the resulting mixture will tend to be "redder". Mixing

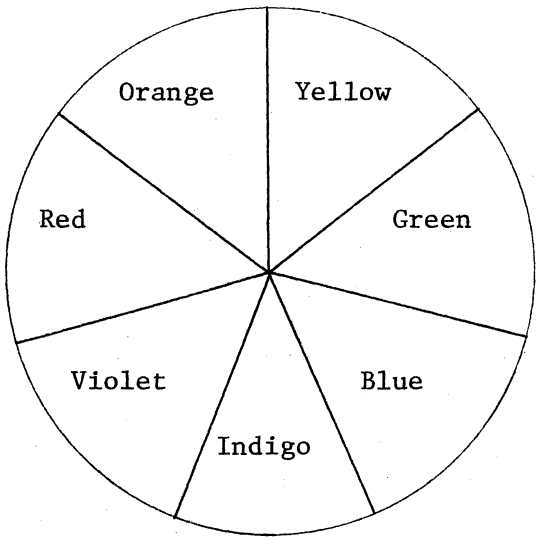


Figure 4. Color circle of sequential color chips (6).

the two colors in equal proportions will produce the intermediate color, orange. Figure 5 illustrates this example.

The line connecting R-Y represents the mixture of red and yellow colors. The point M in between the points R-Y represents the proportions by which the red and yellow were mixed. If a line is drawn from the center, through the point M, the point on the circumference is the resulting color observed (6).

From the implications of the mixture example and Figure 5 the natural conclusion is that the color space as we know it is enclosed by a boundary of pure colors. But the boundary need not be circular in shape. Young (6) suggests that a triangle would prove a better shape for a color-space than the circle.

The three apexes of the triangle represent the three nerves

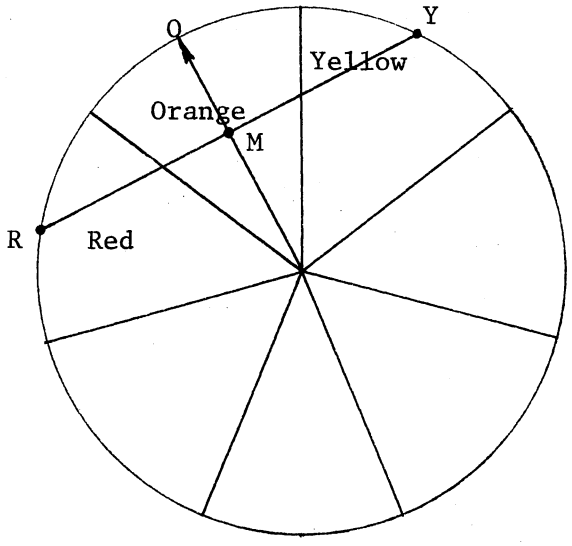


Figure 5. The line connecting R-Y represents the mixture of red and yellow colors. The point M in between the points R-Y represents the proportions by which the red and yellow were mixed. If a line is drawn from the center, through the point M, the point on the circumference is the resulting color observed (6).

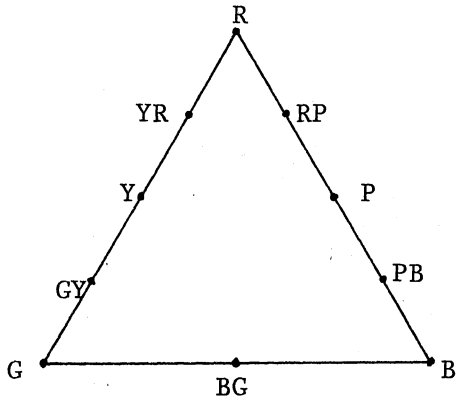
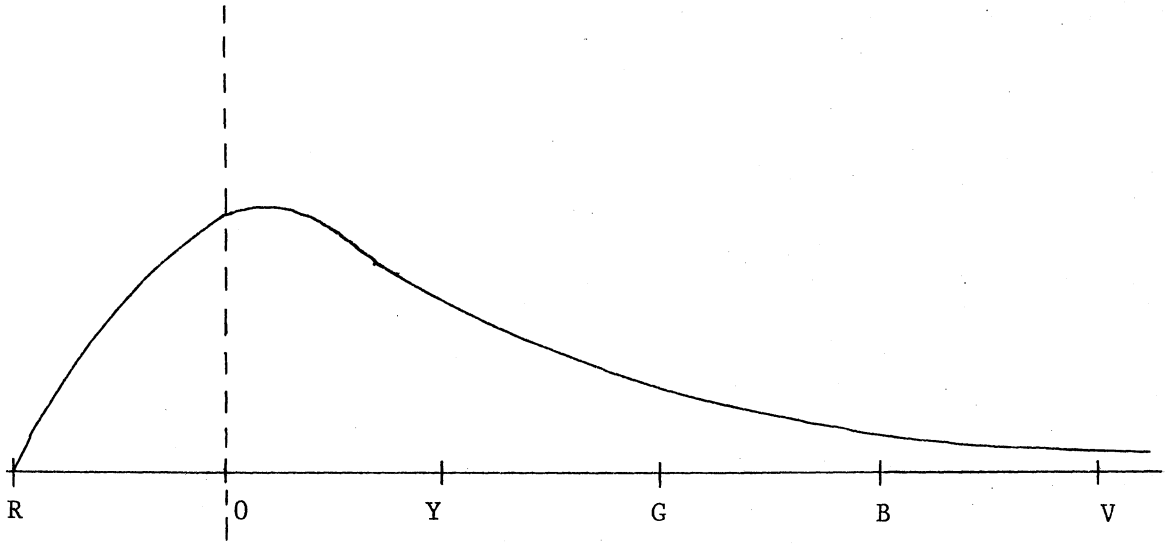
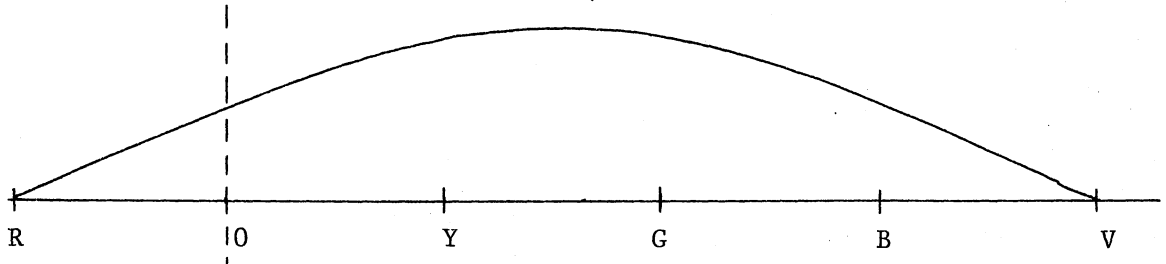


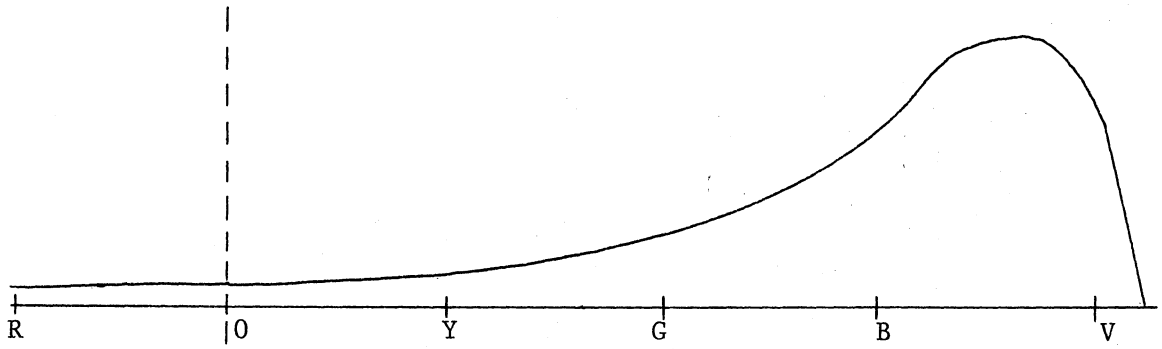
Figure 6. Young's Color Triangle (6).



a) Magnitude of response of the red nerves to light.



b) Magnitude of response of the green nerves to light.



c) Magnitude of response of the blue nerves to light.

Figure 7. The dotted line represents the mixture of the three primaries (red, green, blue) to obtain the color orange seen in Figure 5. (6)

contained in the eye. Excitation of these nerves produce the visual colors. (see the Young-Helmholtz color theory (6)). Figure 7 shows response curves of the three nerves by radiant energy, light. The color orange represented in the color circle diagram, can be seen to be a combination of a large amount of the red wavelength, a medium amount of the green wavelength and a small amount of the blue wavelength. The combination of these three wavelengths is a resulting color, orange (6).

This mixing of the three primaries to form another color can be illustrated as follows. Suppose a disc is divided in half. One half is illuminated by white light and the other half is illuminated by colored lights. The colored lights are to be mixed until a match of the white light is obtained. If a perfect match is obtained, the line dividing the two halves will disappear. Figure 8 represents this illustration (6).

This matching can be described by the following equation

$$W \equiv r(R) + g(G) + b(B) \tag{3.1}$$

where

W	is the white light
r,g,b	the energies of the red, green and blue primaries needed to match the white light
(R) (G) (B)	symbolic representation or red, green and blue.

Colors other than white may also be matched by an appropriate mixture of the three primaries. In the color circle example the orange light may be matched by a mixture of red and green lights, with the amount of blue light reduced to zero. This can be represented by

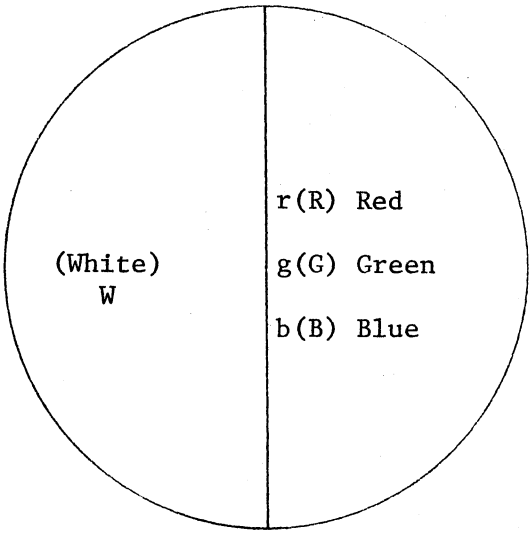


Figure 8. Color matching diagram for white light (6).

Figure 9 and by the equation

$$0 \equiv r(R) + g(G)$$

where

0 is the orange light

r,g the energies of the red and green primarys needed to match the white light.

Now suppose we double the amount of light in the orange sample. To obtain a match to this new sample of orange we must also double the amounts of red and green lights in the mixture of primaries. This is an instance of the distributive law of algebra, which is applicable to color matching experiments (6).

A curious aspect deriving from the mathematical concepts stated above, is that it allows the algebraic negative addition of color. Of course negative color does not exist in reality but its algebraic applications can be used to explain some singularities that arise.

For example, suppose we have a blue-green sample and we wish to

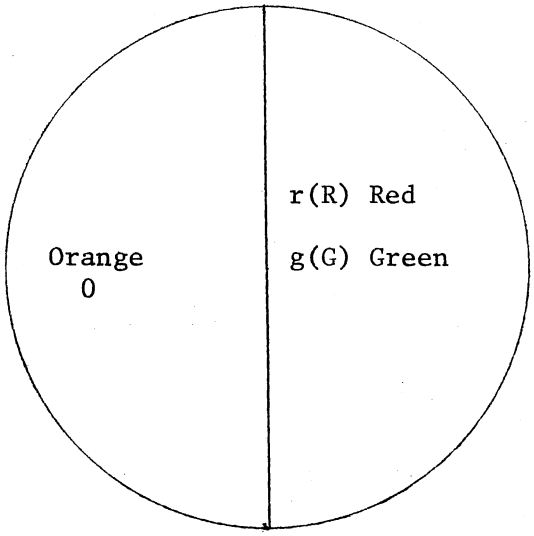


Figure 9. Color matching diagram for orange light (6).

match that sample by a combination of the blue and green primaries. But it is found that no combination of the two primaries ever looks exactly like the blue-green sample.

$$X \dot{\equiv} g(G) + b(B) \tag{3.3}$$

Now, a small amount of red is mixed with the sample and a match is obtained.

$$X + r(R) \equiv g(G) + b(B) \tag{3.4}$$

This addition of red to the sample is algebraically equivalent to subtracting it from the mixture of the blue and green primaries. Thus

$$X \equiv g(G) + b(B) - r(R) \tag{3.5}$$

In general, if we are given four colors, where three of the colors are suitably selected primaries, some combination of the three primaries can be used to match the fourth color. At times, however, one of the three primaries may have to be added to the fourth to obtain apparent equality of the two mixtures (6).

We come now to the description of the C.I.E. system which in most ways uses the concepts of the mixing of red, green and blue primaries as shown above. In general the mixing of the three primaries can be represented by the equation

$$c(C) \equiv r(R) + g(G) + b(B)$$

where

(C) is the resulting color of the mixture of the three primaries red, green and blue.

(R) represents the symbol for red

(G) represents the symbol for green

(B) represents the symbol for blue

r is the amount of red radiant energy

g is the amount of green radiant energy

b is the amount of blue radiant energy

c is r + b + g

It has become the practice to base the units of (R), (G), and (B) on a match of white light of some defined quality. Since white may be regarded as a color in which neither red nor green nor blue predominates, the convention that equal amounts of (R), (G), and (B) are required to match white may be adopted. We can, therefore, visualize a color-measuring system in terms of the red, green and blue stimuli of some trichromatic spectrometer with the units of the stimuli adjusted to be equal in a match to a standard white (5).

> "Although a manufacturer may be primarily interested in the color of his product viewed by daylight, it serves no useful purpose to check it against the standard color for all phases of daylight. These phases might include light from the clear sky, light from the sun plus clear sky, light from the overcase sky, direct sunlight, or mixtures of these with light reflected from a brick wall, and so forth. To permit color measurements to be directly comparable as often as possible it has become customary

> to use one of three standard light sources. These standard sources were recommended in 1931 by the Commission Internationale de l'Eclairage (CIE)" (5)

It is recommended that the following three illuminants be adopted as standards for the general colorimetry of materials (9):

Illuminant A.

A gas-filled lamp of color temperature 2848 K.

Illuminant B.

The same lamp used in combination with a filter composed of a layer one centimeter thick of each of two solutions B_1 and B_2, contained in a double cell made of non-selective optical glass.

Illuminant C.

The same lamp used in combination with a filter consisting of a layer one centimeter thick of each of two solutions, C_1 and C_2, contained in a double cell made of non-selective optical glass.

It is recognized that for certain special applications (for example, the specification of signal glasses) other luminous sources may be prescribed, but in the absence of special conditions one of the three indicated sources should be used.

Illuminant A is intended to be typical of light from the gas-filled incandescent lamp, Illuminant B is an approximate representation of noon sunlight, and Illuminant C is an approximate representation of average daylight (9).

By addition (or algebraic subtraction) of amounts of three primary lights, as described earlier, we can match any test light. All the spectrum lights can be matched by combining positive and negative amounts of the three primary lights. For points of reference choose

700 nm red, 546 nm green, and 436 nm blue. Figure 10 shows the relative amounts of spectral energy needed by a person with normal color vision to match any of the spectrum colors, provided each spectrum light source emits the same amount of energy.

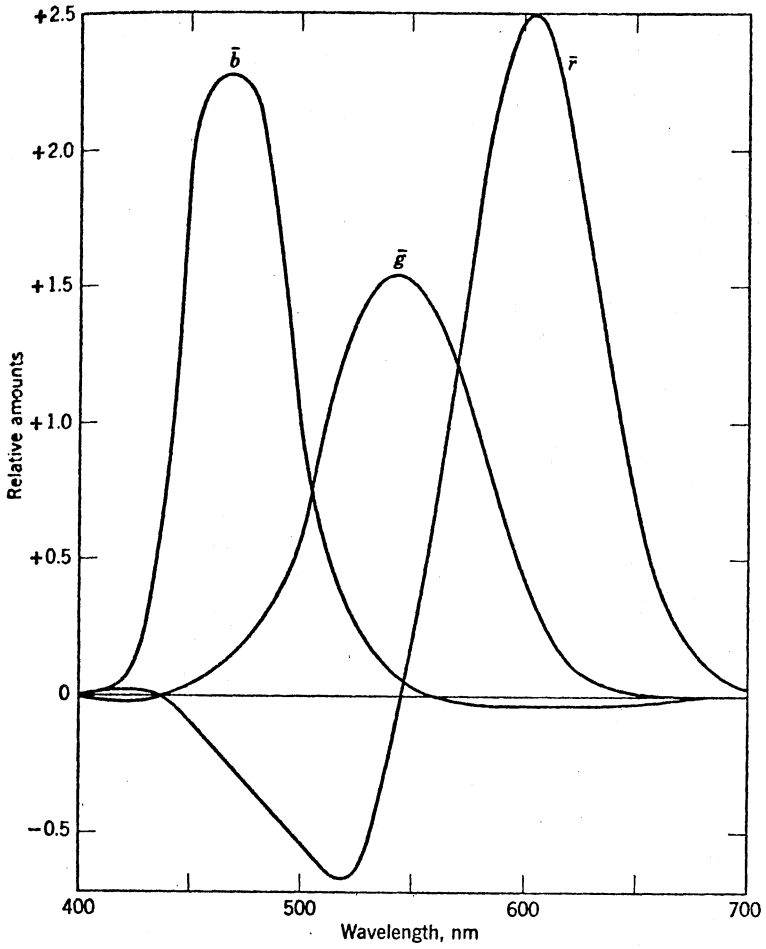


Figure 10. These curves show, for each wavelength the amounts r,g, and b of the 700 nm red R , 546 nm green C , and 436 nm blue B primaries needed by a normal observer to match each of the colors of the equal-energy spectrum (CIE 1931 R - G - B system) (2).

Since we will ultimately be interested in objects instead of just lights, let us generate a test color X by shining the light, in turn, from each of the standard illuminants A,B, and C through a transparent colored object onto one side of a disk. Let us put an entire series of spectrum lights on the other side of the disk (spaced 10 nm apart, across the spectrum from 400-7000nm). All the spectrum lights are to have the same energy applied to them. Now we adjust the energy of each spectrum light until it equals the energy for the standard illuminant and object at each of the spectrum light wavelengths. When this happens we have, at 10 nm intervals, the energy distribution curve reaching the observer from the standard illuminants and the object (2). Figure 11 shows this illustration.

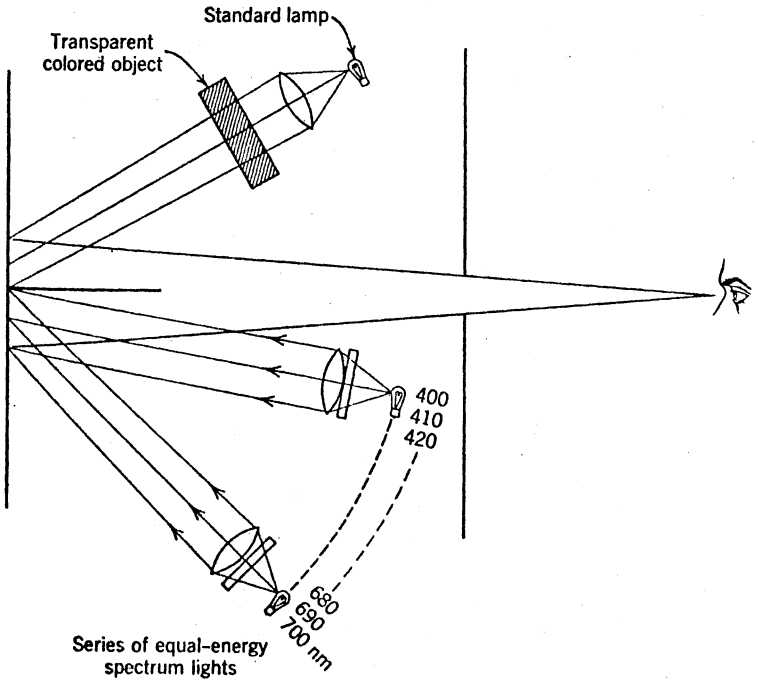


Figure 11. Arrangement for using spectrum lamps to duplicate the spectral energy distribution of a source and a transparent object (2).

Figure 12 shows how we can use this information to estimate the energy distribution reaching the observer in Figure 11.

Now, replace the standard illuminant lamp with the three primary lamps (R) red, (G) green, and (B) blue, and repeat the previous exercise but without the transparent colored object. Do not adjust the spectral lamps, but adjust the three primary lamps. This will give tristimulus values r,g, and b which are correct for the test object and standard illuminant lamps. Figure 13 illustrates this exercise (2).

If each spectrum lamp had unit energy, it would take an amount of light $\bar{r}$ from the red light, plus amount $\bar{g}$ from the green light, and amount $\bar{b}$ from the blue light to match the color of the transparent object illuminated by one of our three standard illuminants. But each spectrum lamp has energy ExT, therefore it will take amounts $\bar{r} \times E \times T + \bar{g} \times E \times T + \bar{b} \times E \times T$ to match our object. Thus we can find tristimulus values if we know the spectral energy distribution of the standard illuminant, the spectral transmittance curve of the illuminated object and the color matching functions $\bar{r}$, $\bar{g}$, and $\bar{b}$ of the normal observer. Figure 14 illustrates this. (See (3) for tables of the $\bar{r}$,$\bar{g}$ and $\bar{b}$ functions.)

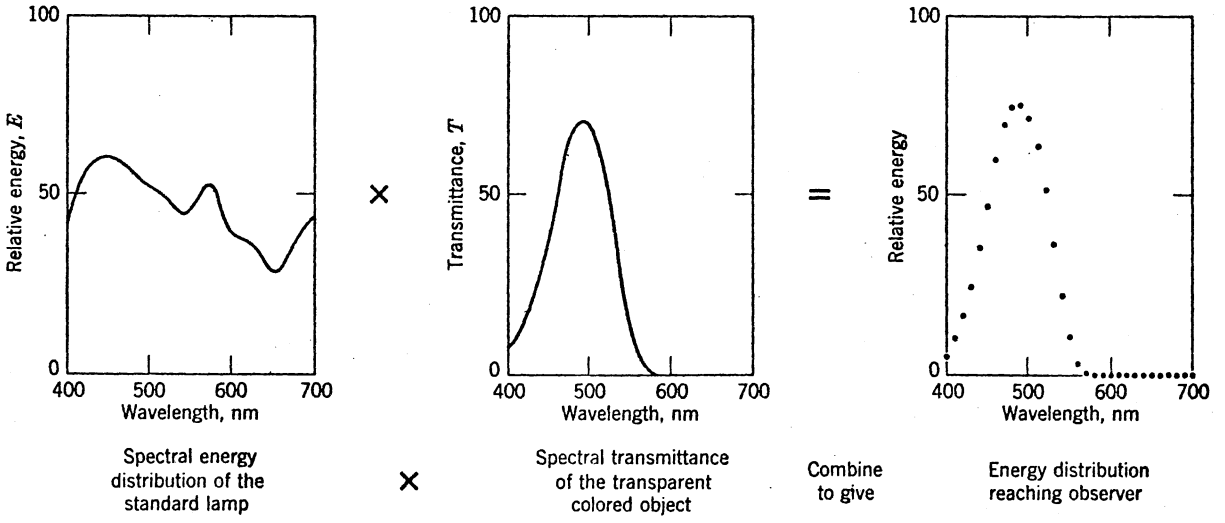
It should be clear that tristimulus value r is equal to the sum over all wavelengths of $E \times T \times \bar{r}$ and similarily for g and b. In mathematical terms, (3)

$$r(R) = \sum_{\lambda} ET\bar{r} \quad \text{or} \quad \int_{\lambda} ET\bar{r}d\lambda \tag{3.6}$$

$$g(G) = \sum_{\lambda} ET\bar{g} \quad \text{or} \quad \int_{\lambda} ET\bar{g}d\lambda \tag{3.7}$$

$$b(B) = \sum_{\lambda} ET\bar{b} \quad \text{or} \quad \int_{\lambda} ET\bar{b}d\lambda \tag{3.8}$$

The only problem with the aforementioned system is that it can generate negative numbers. To alleviate this problem the C.I.E. proposed



At each wavelength, the amount of light coming to the observer is obtained by multiplying E times T: $E \times T = ET$.

Figure 12. Diagram of relative energy reaching the eye of a normal observer. (2)

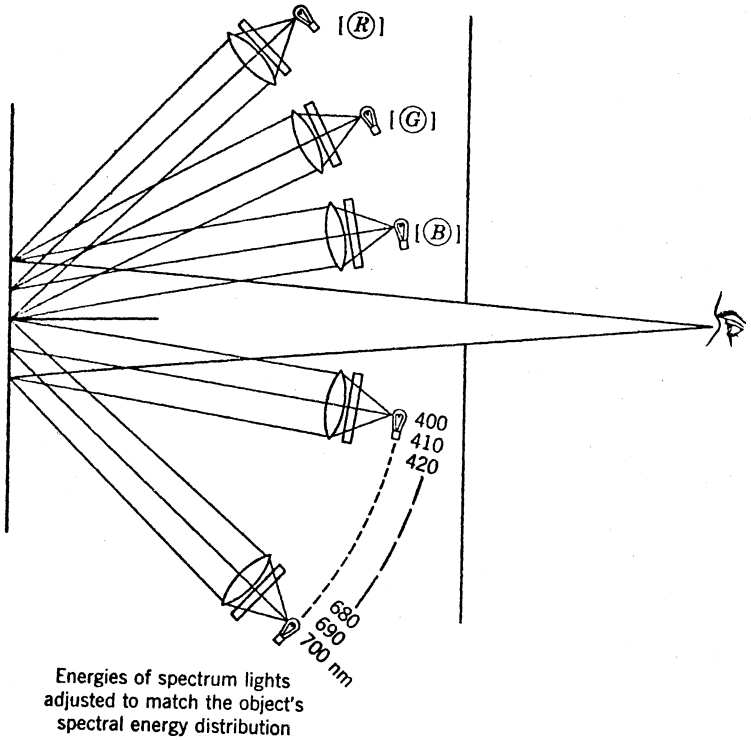


Figure 13. Energies of spectrum lights adjusted to match the object's spectral energy distribution (2).

the following solution (9). The C.I.E. selected a particular set of primaries. These primaries were chosen so that:

1) all possible real colors could be matched by positive amounts of the primaries, so that negative numbers did not appear in the C.I.E. system

2) a relatively large range of colors in the yellow-red region could be matched with only two primaries

3) the intensity of the light needed to make the match (lightness of the color) could be specified by one of the primaries alone.

Thus, if the 1931 C.I.E. imaginary primaries are called X,Y,Z, then the X,Y, and Z values can be calculated from the RGB (red, green,

blue system) in the following way (3).

For (R): Let X = 2.7689 ; Y = 1 ; Z = 0 (3.9)

For (G): Let X = 0.38159 ; Y = 1 ; Z = 0.012307 (3.10)

For (B): Let X = 18.801 ; Y = 1 ; Z = 93.066 (3.11)

Knowing (luminous transmittance of each of the primaries. L=1,2,3) then:

$$(R): \quad X = 2.7689L_1;\ Y = L_1;\ Z = 0 \tag{3.12}$$

$$(G): \quad X = 0.38159L_2;\ Y = L_2;\ Z = 0.012307L_2 \tag{3.13}$$

$$(B): \quad X = 18.801L_3;\ Y = L_3;\ Z = 93.066L_3 \tag{3.14}$$

Therefore

$$X = 2.7689L_1 + 0.3815L_2 + 18.891L_3 \tag{3.15}$$

$$Y = L_1 + L_2 + L_3 \tag{3.16}$$

$$Z = 0.012307L_2 + 93.066L_3 \tag{3.17}$$

The above system is used for transforming the RGB system into the XZY system. From the coordinates $\bar{r},\bar{g}$, and $\bar{b}$ the corresponding coordinates $\bar{x},\bar{y}$, and $\bar{z}$, can be calculated using the relationship above. Figure 15 shows the graph of the $\bar{x},\bar{y}$, and $\bar{z}$ functions.

It should immediately be seen from the comparison of Figures 10 and 15 that the objection of negative values in the RGB system has disappeared in the XYZ system.

The XYZ tristimulus values can be represented in the following mathematical forms (3)

$$X = \sum_{\lambda} EL_1\bar{x} \quad \text{or} \quad \int_{\lambda} EL_1\bar{x}d\lambda \tag{3.18}$$

$$Y = \sum_{\lambda} EL_2\bar{y} \quad \text{or} \quad \int_{\lambda} EL_2\bar{y}d\lambda \tag{3.19}$$

$$Z = \sum_{\lambda} EL_3\bar{z} \quad \text{or} \quad \int_{\lambda} EL_3\bar{z}d\lambda \tag{3.20}$$

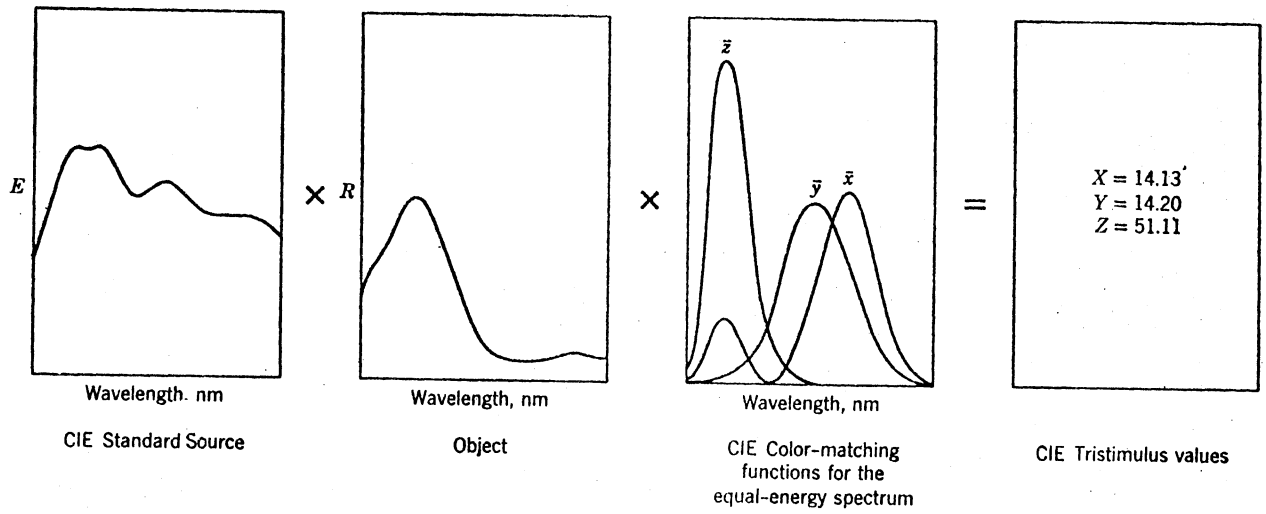


Figure 14. The CIE tristimulus values X,Y, and Z of a color are obtained by multiplying together the relative energy E of a CIE standard light source, the reflectance R (or the transmittance) of the object, and the tristimulus values of the equal-energy spectrum colors $\bar{x}$,$\bar{y}$, and $\bar{z}$. The products are summed up for all the wavelengths in the visible spectrum to give the tristimulus values, as indicated in the diagrams and by the mathematical equations (2).

In discussing the perception of color it is important to keep in mind the distinction between color and brightness. The color of an object in a room does not change significantly if more lamps are turned on. The object may appear to be brighter, but it will still be the same color. Therefore the three numbers needed to specify a color should be independent of the absolute energy reflected by an object. Howevever, the absolute values of X,Y, and Z do change when light energy is changed.

Consequently, it is desirable to express the three numbers used to describe a color in terms of proportions or relative amounts. The XYZ primaries were so chosen that a white light comprised of all wavelengths of equal energy could be matched by equal values of X,Y, and Z. The following formula makes it possible to say that X,Y, and Z each contributes one-third of the energy of a mixture that duplicates the apparent white color of the object even though the amount of light reflected or transmitted by the object varies from one situation to another.

$$\frac{X}{X+Y+Z} + \frac{Y}{X+Y+Z} + \frac{Z}{X+Y+Z} = 1 \tag{3.21}$$

Now let

$$\frac{X}{X+Y+Z} = x \tag{3.22}$$

$$\frac{Y}{X+Y+Z} = y \tag{3.23}$$

$$\frac{Z}{X+Y+Z} = z \tag{3.24}$$

Then

$$x+y+z = 1 \tag{3.25}$$

It follows from this that if the values of x,y are known, then the value of z is determined

$$z = 1 - x - y \tag{3.26}$$

With these relationships it is possible to plot the effects of mixing the standard x,y,z primaries in all possible combinations in a two-dimensional graph (3).

The strange-looking graphs of Figure (16) are known as chromaticity diagrams. All four graphs can be used in color calculations, but most of the time graph (d) is used (3). The coordinates for white light (w) are located around the center of gravity of the system, approximately (0.3333 0.3333). This will vary, but only slightly, for each illuminant A,B, and C.

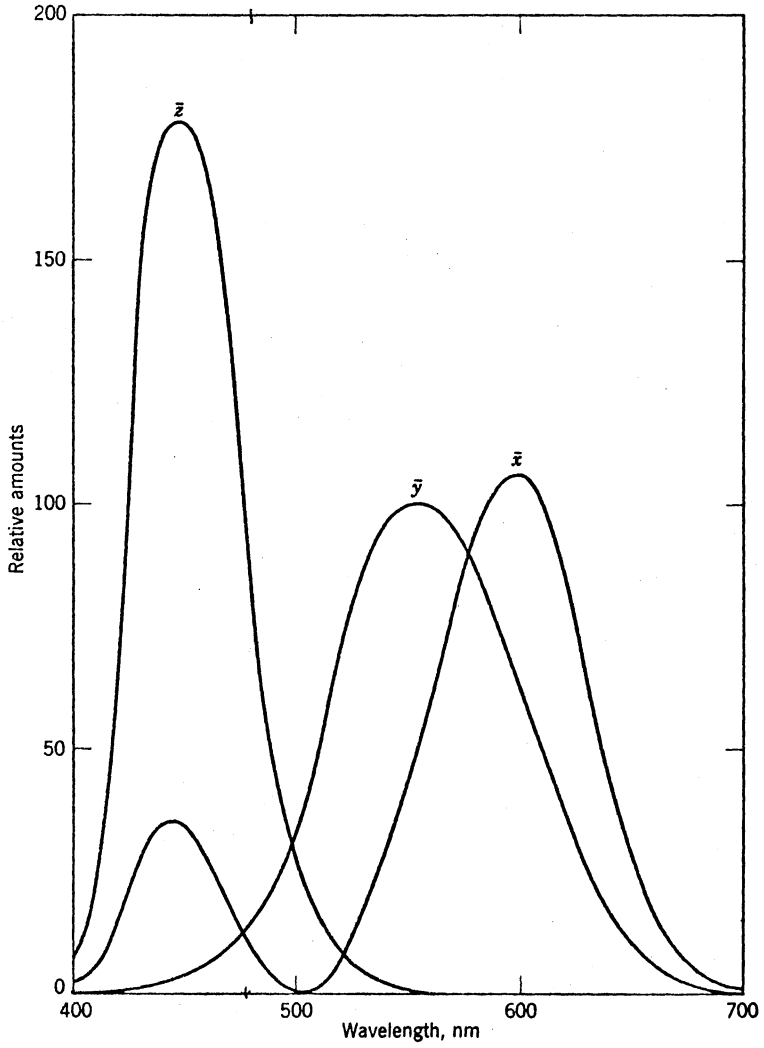


Figure 15. The CIE color-matching functions x,y, and z for the equal-energy spectrum are also the tristimulus values of the equal-energy spectrum colors (2).

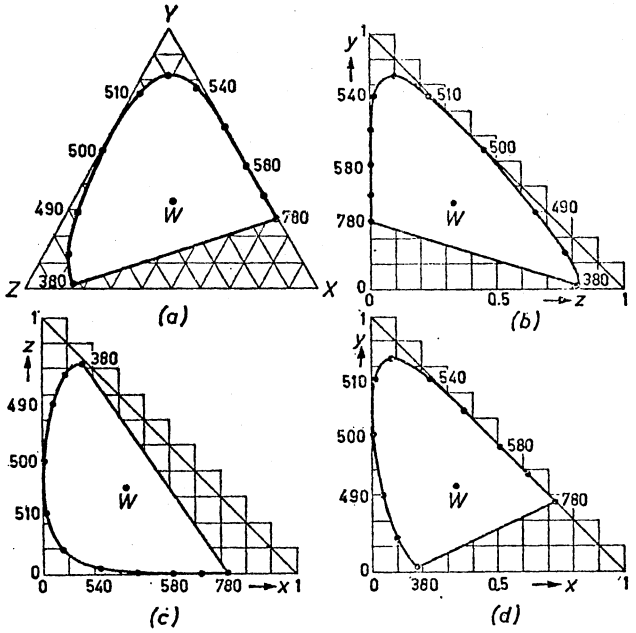


Figure 16. Chromaticity diagrams of colour triangles derived from the C.I.E. 1931 XYZ system (2).

Figure 16.

a) Equilateral form. XYZ

b) Rectangular form. y-z

c) Rectangular form. z-x

c) Rectangular form. y-z

The curved line in Figure 17 is called the locus of spectral colors. The points on this locus represent the values of x,y, and z needed to produce spectrally pure colors. Point E is the equal-energy white point (0.3333, 0.3333). Point S_A is the illuminant A, point (0.4476, 0.4075). Point S_B is the illuminant B, point (0.3485, 0.3517) and point S_C is the illuminant C, point (0.3101, 0.3163) (5).

There are a number of features to be noted about the spectrum locus. First, it should be seen that from the extreme red end of the spectrum, 700 nm, to the green wavelength of about 530 nm, the locus is nearly a straight line. The locus then bends sharply around as the color changes from green to blue-green 510 nm, the blue-green wavelengths from 510 nm to 480 nm are spread over the locus which has a slight curvature, while the blue-violet wavelengths are compressed into a very small tail.

This is primarily a function of the receptor nerves in the eye. A second important characteristic is that the locus is always either straight or convex, but never concave. From this it should be seen that the color resulting from the mixture of any two wavelengths must be either on the locus or within the area bounded by the locus, but never outside. Hence the area bounded by the locus and the straight line joining its red and violet extremities defines the region of the chromaticity chart outside which no physical stimulus will be located.

The preceding information was intended to give the reader some idea of the C.I.E. reference system. It is not the only reference system available today, but because of its simple structure it is the one most often used. The C.I.E. system is utilized in the Munsell color system, which is described in Chapter IV. Figure 18 is a color diagram of the spectrum locus.

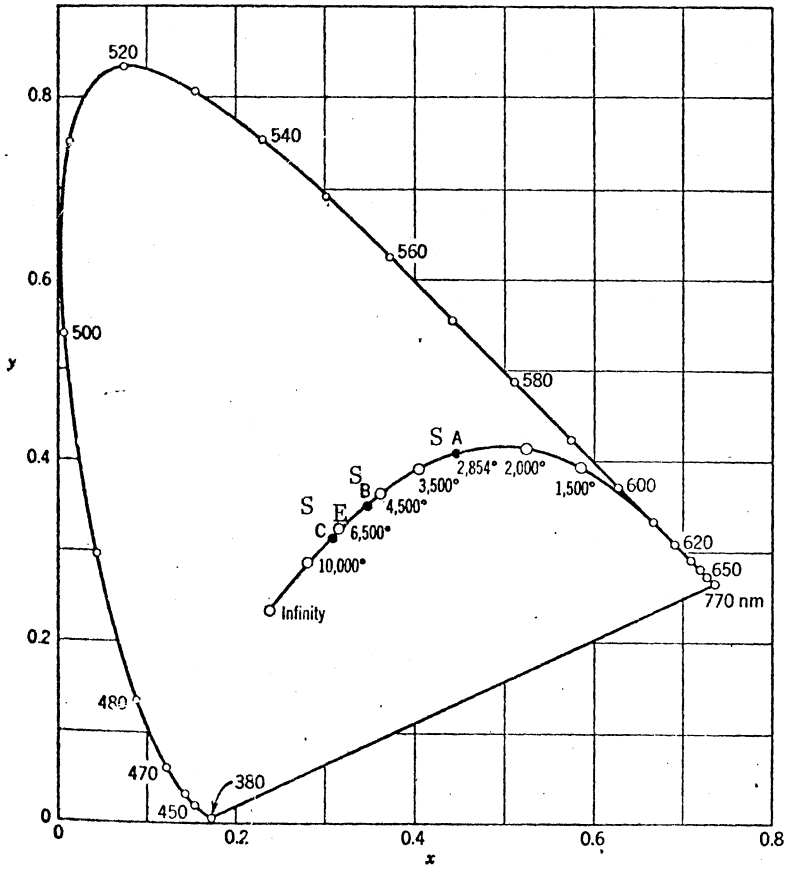


Figure 17. Spectrum locus and standard illuminants.

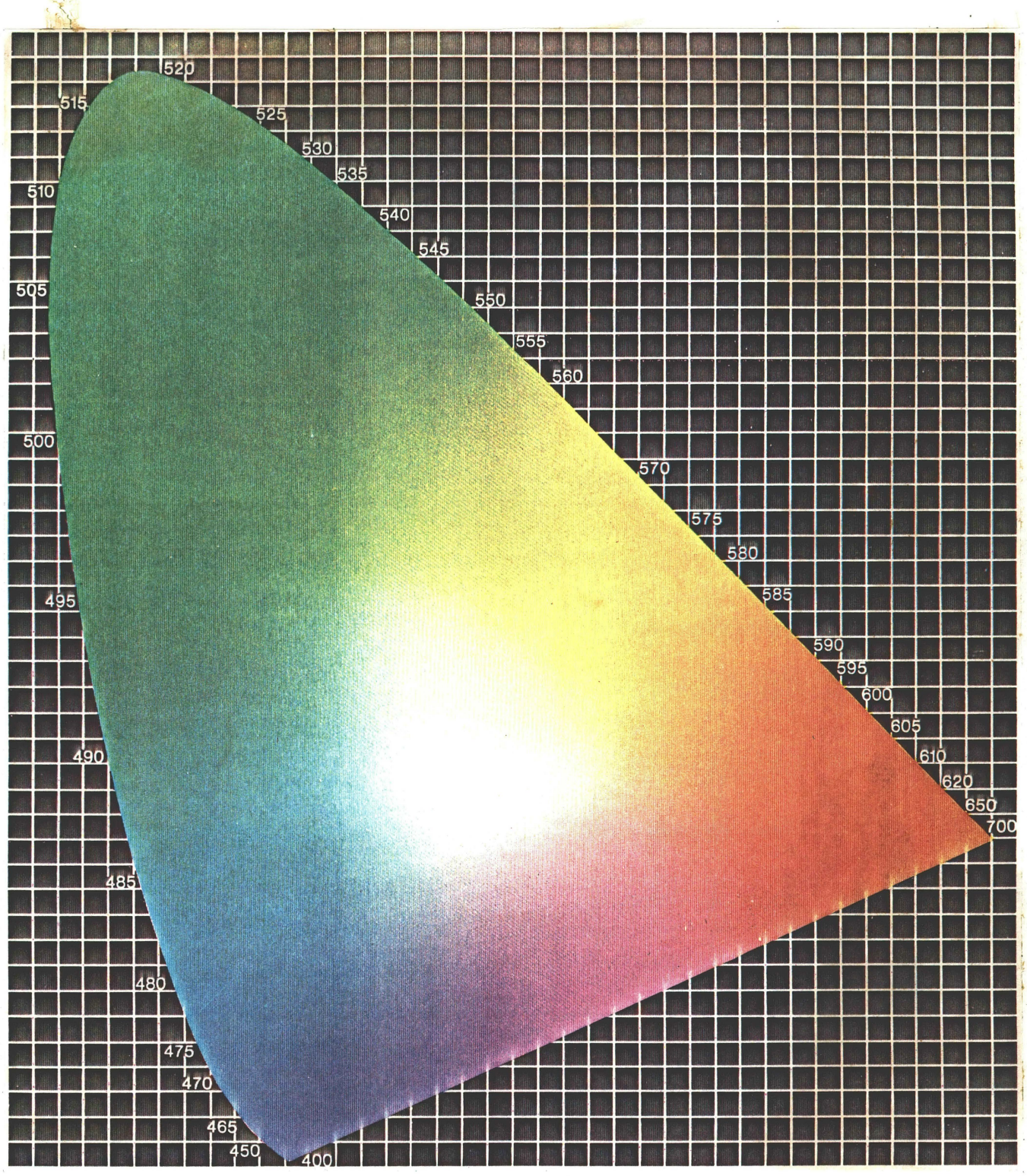


Figure 18. Color print of spectrum locus. (15)

CHAPTER IV

THE MUNSELL COLOR SYSTEM

Albert Henry Munsell (1859-1918) is preeminent among American colorists. Born in Boston, he devoted his life to art, studied widely in Europe and became senior instructor at the Massachusetts Normal Art School. In 1900, at the age of forty-one, he developed and patented his famous Color Sphere. It was immediately accepted as one of the most intelligent systems of color organization ever achieved. Today, the standards he set are used as a practical method of color designation.

Munsell reasoned that the color appearance of objects was apparent only in objects and so proposed to set up a series of colored papers which varied equally in all directions in the color domain of visual appearance. In theory, at least, he was not concerned with the physical variables involved, but only with the color as such, viewed under more or less standardized conditions. With this in mind he and his associates succeeded in producing a color atlas containing examples of colors which varied in roughly uniform steps over a large part of the spectrum of visible colors (3).

He defined the three variables of color as hue, value, and chroma. These he constructed into a three-dimensional color solid having a vertical black-to-white axis. Around this he arranged the hues in equal angular spacing and defined chroma as the distance from the axis at any particular value level. His published atlas contains colored paper chips

arranged according to various sections through his solid, some passing at right angles to the value axis, some including this axis in the plane.

In use it was intended that the sample to be specified should be held near the atlas and the position of the sample with respect to the actual chips determined. The specification of the color then consisted of a numerical statement of its position on the scales of the solid (13).

The Munsell method of color notation may be used directly if measurements are made by comparison to Munsell charts, indirectly by converting I.C.I. XYZ notation into Munsell notation. In the Munsell notation, color is expressed in units of visual difference of the three psychological attributes: Hue, Value (lightness), and Chroma (saturation) (13).

Munsell hue is that attribute of certain colors in respect to which they differ characteristically from a gray of the same lightness and which permits them to be classed as reds, yellows, greens, blues, or purples. The Munsell hue circuit is divided into 10 major hues, as follows:

R:	red	BG:	blue-green
YR:	Yellow-red	B:	blue
Y:	yellow	PB:	purple-blue
GY:	green-yellow	P:	purple
G:	green	RP:	red-purple

The letters to the left of the colon are used to represent the hue which is to the right of the colon. The above hues are shown in Figure 19. Each of the 10 major hues is equally spaced visually (when held constant for value and chroma), and the circuit is divided into 100 closer but still equally spaced hues in order to provide a numerical decimal notation as fine as may be needed for use in the most careful measurement problems.

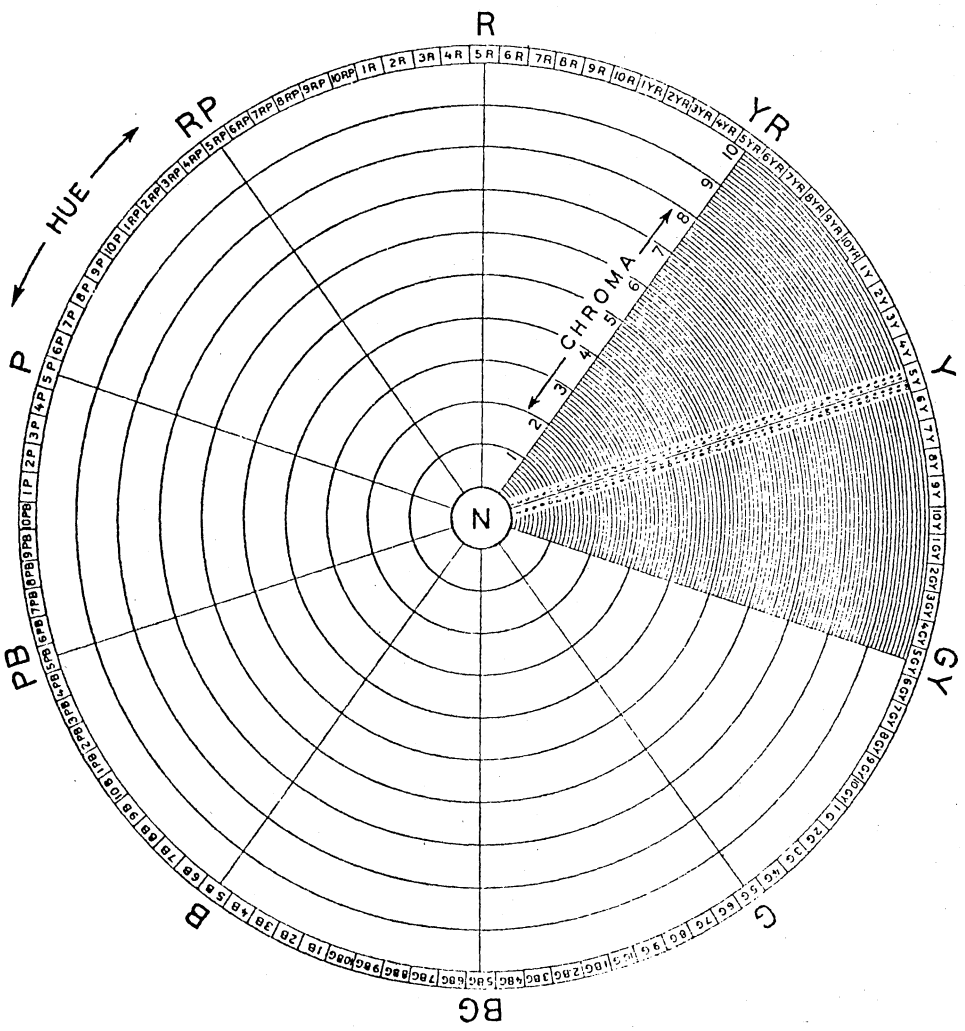


Figure 18. Munsell hue color notation (13).

Munsell value is that attribute of all colors which permits them to be classed as equivalent to some member of a series of grays that are equally spaced under the standard conditions for which the scale was derived. The Munsell scale of grays extends from 0, (black), to 10, (white), and the use of decimal numbers permits the value notation to be expressed as accurately as seems necessary for the prupose at hand. Figure 20 shows the value scale used in the Munsell system (13).

Munsell chroma is that attribute of all colors possessing hue which determines their degree of difference from a gray of the same value. The notation is numerical, with 0 at gray, extending outward from the neutrals toward 10 for the strong colors. The chroma

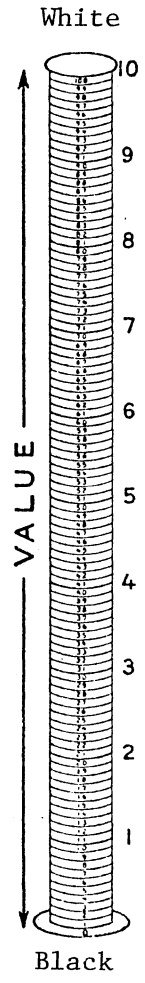


Figure 20. Munsell value axis (13).

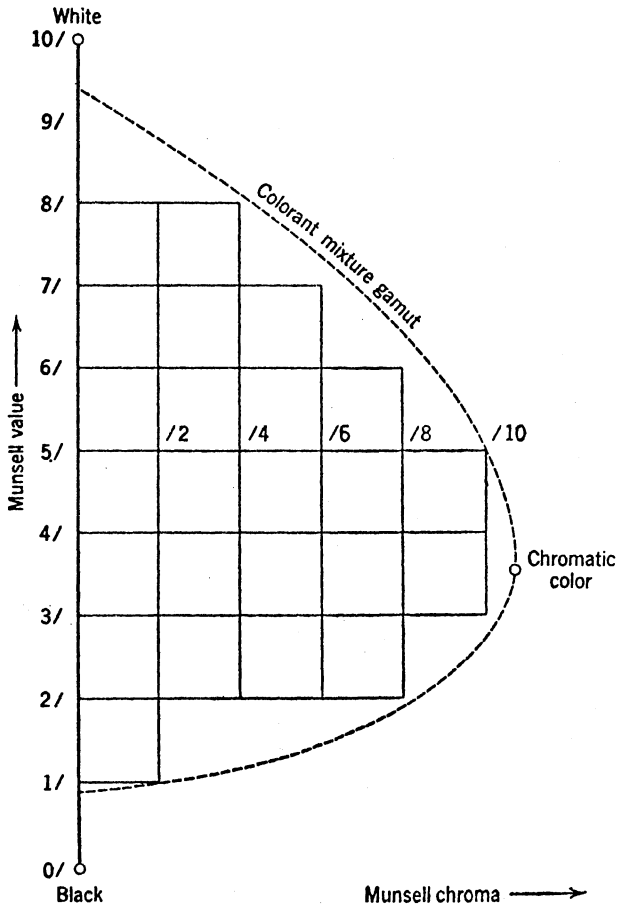


Figure 21. Diagram of Munsell (10).

notation may be subdivided decimally; hence it allows a notation for as fine a degree of chroma difference as may be discriminated. Figure 21 shows how chroma may be visualized. Figure 22 places all three concepts of hue, value, and chroma together (13).

It is important in color work to understand thoroughly the three-dimensional concept of color that is illustrated in Figure 22. The three attributes of color can be treated as quantities and specified numerically, if all discriminable colors are conceived to be arranged in a system such that neighboring members differ from each other in each of

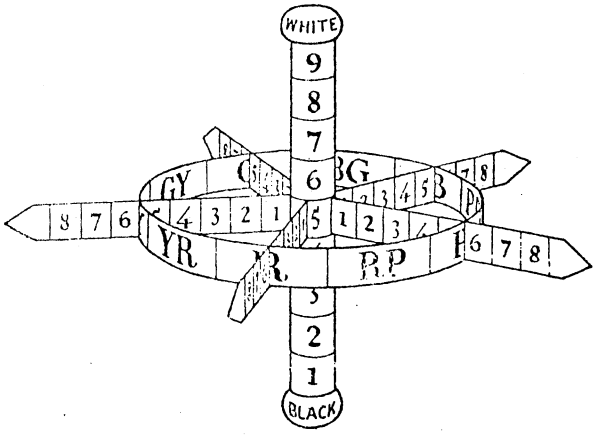


Figure 22. Munsell's three dimensional concept of color (13).

the three attributes by just noticeable degrees. Such a system is necessarily three-dimensional, and three ordinal values representing the positions of a given color in the three dimensional space are needed to define the color (13).

The Munsell notation is written in the order Hue Value/Chroma or H V/C. Therefore, 10R 5/10 would represent:

1) a hue of color red. The location of the hue on the hue circle would be in the red segment, subdivision 10.

2) a value of 5. The location of the value on the vertical value axis would be mid-way between value 0 (black), and value, 10, (white).

3) a chroma of 10. The location of the chroma would be away from the value axis, 10 chroma steps.

Figure 23 gives an illustration of the Munsell color 10R 5/10. The color is represented by the hashed portion of the three dimensional diagram.

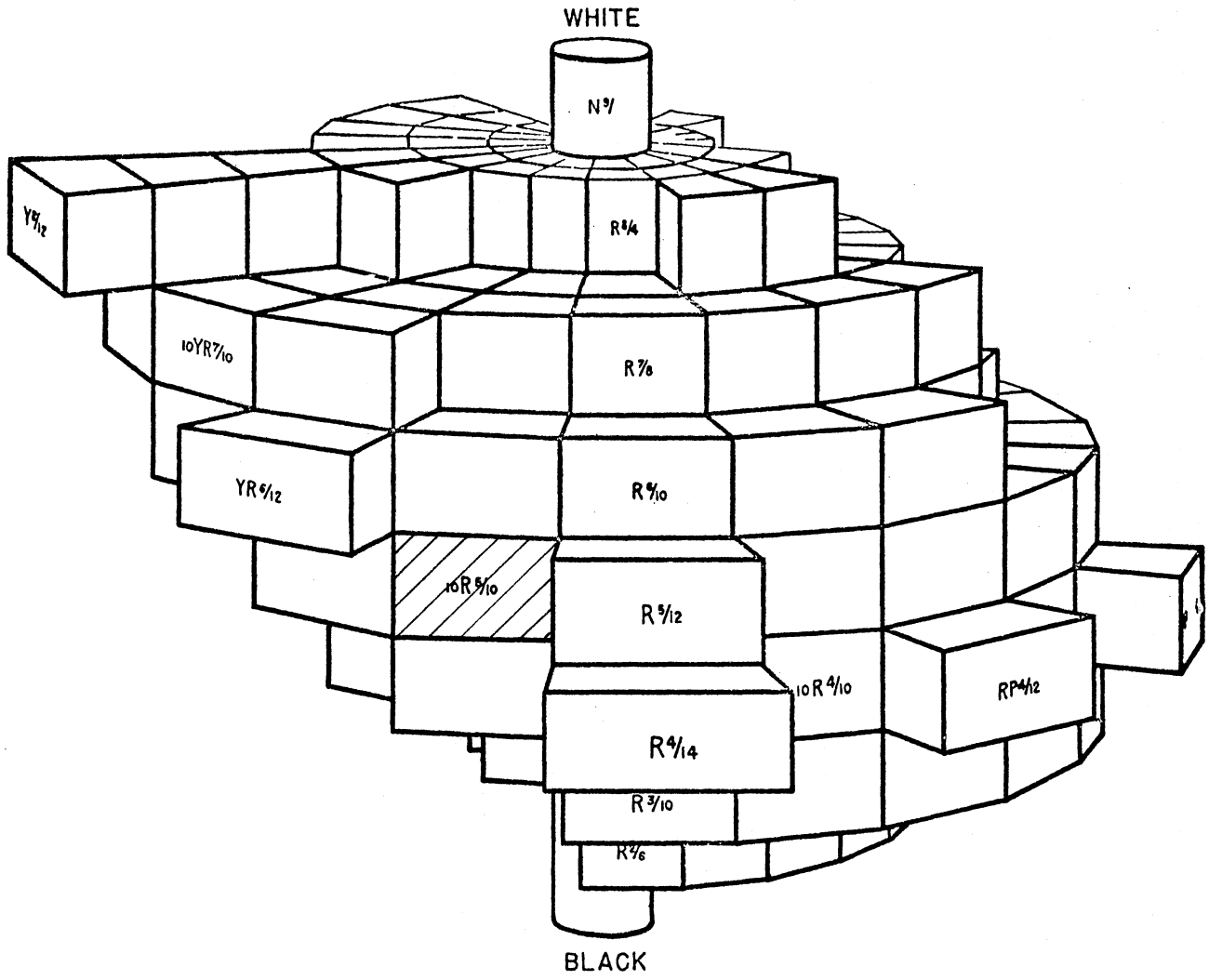


Figure 23. Diagram to illustrate Munsell hue, value and chroma in their relation to one another (13).

Of course, with the subdivisions of the system a finer discrimination can be obtained. For example 9.6R 4.8/9.5would be a different color than that of 10R 5/10 but to the human eye the distinction may not be possible.

In 1931 the ICI committee on illumination recommended that all subsequent color data be expressed in terms of the same tristimulus system, so that the results would be immediately comparable. The standard observer and coordinate system recommended was the XYZ system. The green primary chosen, designated by Y, is such as to carry all the luminosity. The other two primaries (red, blue), whose amounts are designated by X and Z, respectively, being unassociated with luminosity. It is convenient and customary to express the Y value in terms of luminous directional reflectance. Luminous directional reflectance refers to the flux reflected in the direction of the observer and is the reflectance that a perfectly diffusing surface illuminated and viewed like the specimen would need in order to attain the same luminance as the specimen. Luminous directional reflectance is closely related to the appearance of the specimen, and is therefore the more commonly used as an evaluation of Y for opaque specimens (11).

The reflectance standard recommended for the colorimetry of opaque specimens is a white surface prepared by collecting on a metallic or other suitable surface an opaque layer formed by the smoke from magnesium ribbon burning in air. It is now nearly universal practice to carry out the spectrophotometry of color standards relative to the magnesium oxide standard (11).

Along with Y, the x,y chromacity values need to be calculated in order to convert from XYZ notation to the Munsell notation. The x,y calculations were discussed in Chapter III. Figure 24 gives a visual

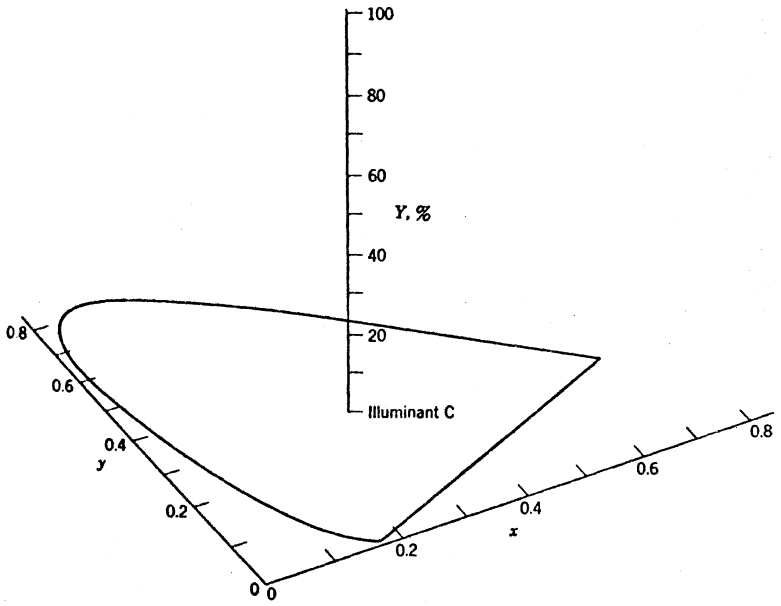


Figure 24. Diagram of x,y chromaticity and luminance Y value.

combination of the Y, x,y values. The third dimension of color is conveniently added to the CIE chromaticity diagram by thinking of the luminance or lightness axis rising up from it. Lighter colors lie in space at their proper level of Y and above the point representing their chromaticity x,y (11).

Munsell correlated his value V with the ration Y_r, where Y_r is the luminance factor of the sample Y divided by the luminance factor Y_{MgO} of magnesium oxide. This quantity can be measured easily with physical photometric equipment. As $Y_{MgO} = 0.975$, $Y_r = Y/Y_{MgO}$ for the perfect diffuser amounts to 1.0256. A scale of ten apparently equal-value V steps was painted and then by photometric measurement it was found that roughly

$$V^2 = Y_r \times 100 \quad . \tag{4.1}$$

Table II shows the empirical relation between V and Y_r, where for complete black $Y_r = 0$ and for pure white $Y_r = 1.0256$. A good approximation is given by the fifth-degree formula (10).

$$100Y_r = 1.2219V - 0.23111V^2 + 0.23951V^3 - 0.021009V^4 + 0.0003404V^5 \quad . \quad (4.2)$$

TABLE II

MUNSELL VALUE STEPS V AND RELATIVE LUMINANCE FACTORS Y_r WITH RESPECT TO MAGNESIUM OXIDE M_gO

V	0	1	2	3	4	5	6	7	8	9	10
$100Y_r$	0	1.21	3.13	6.56	12.00	19.77	30.05	43.06	59.10	78.66	102.56

Therefore, given the Y_r calculation of an object the corresponding Munsell value V may be obtained by using equation (4.2). The empirical relationship between Y_r and V using equation (4.2) is given in Table III Figure 25 shows a graph of equation (4.2). With Y_r known, Munsell value V can be obtained by interpolation in Table III. Once the value V is known, the hue and chorma may be spotted on the corresponding value chart. Chromaticity charts x-y for Munsell values 1 through 9 are shown in Figure 26 through 34 inclusive. The illuminant used for these charts is standard illuminant C. ($x_w = 0.3101$, $y_w = 0.3163$).

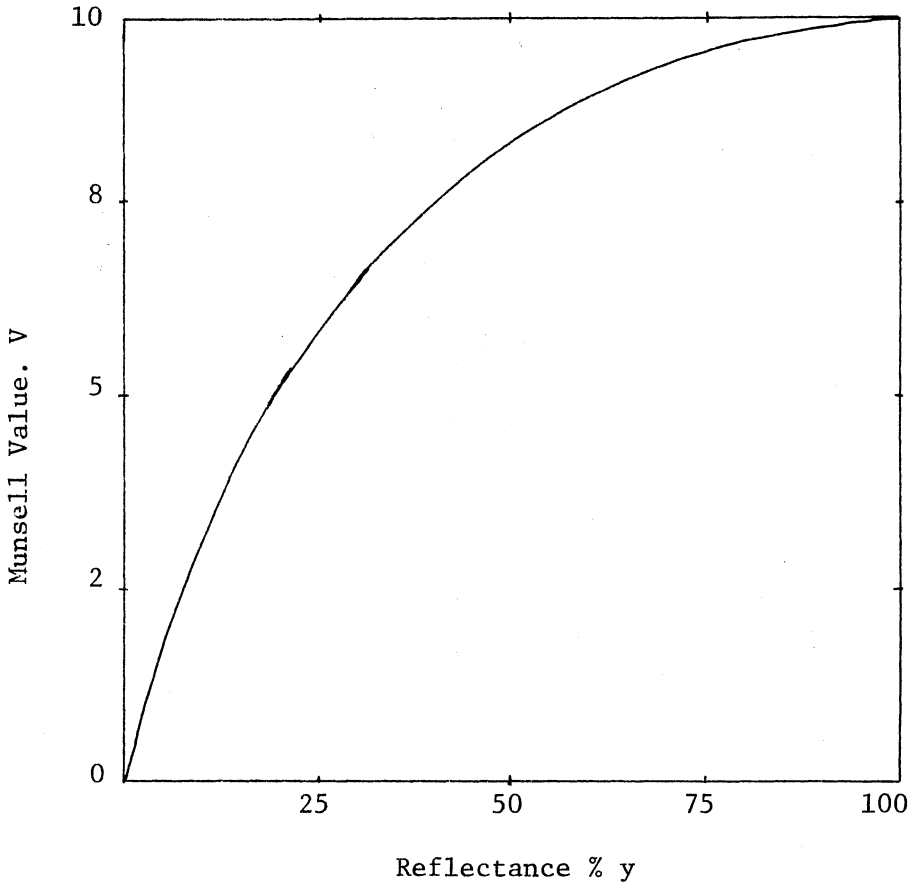


Figure 25. How the Munsell Value Function varies with luminance Y (10).

TABLE III

I.C.I. (Y) EQUIVALENTS (IN PERCENT RELATIVE TO M_gO) OF THE RECOMMENDED MUNSELL VALUE SCALE (V) FROM 0/ TO 10/ (13)

V	Y_V(%)	V	Y_V(%)	V	Y_V(%)	V	Y_V(%)	V	Y_V(%)	V	Y_V(%)	V	Y_V(%)	V	Y_V(%)	V	Y_V(%)	V	Y_V(%)
10.00	102.56																		
9.99	102.30	8.99	78.45	7.99	58.92	6.99	42.92	5.99	29.94	4.99	19.68	3.99	11.935	2.99	6.511	1.99	3.100	0.99	1.196
8	102.04	8	78.23	8	58.74	8	42.77	8	29.82	8	19.59	8	11.870	8	6.468	8	3.075	8	1.182
7	101.78	7	78.02	7	58.57	7	42.63	7	29.71	7	19.50	7	11.805	7	6.425	7	3.050	7	1.168
6	101.52	6	77.80	6	58.39	6	42.49	6	29.59	6	19.41	6	11.740	6	6.382	6	3.025	6	1.154
5	101.25	5	77.59	5	58.22	5	42.34	5	29.48	5	19.32	5	11.675	5	6.339	5	3.000	5	1.141
9.94	100.99	8.94	77.38	7.94	58.04	6.94	42.20	5.94	29.36	4.94	19.23	3.94	11.611	2.94	6.296	1.94	2.975	0.94	1.128
3	100.73	3	77.16	3	57.87	3	42.06	3	29.25	3	19.14	3	11.547	3	6.254	3	2.950	3	1.114
2	100.47	2	76.95	2	57.69	2	41.92	2	29.13	2	19.06	.2	11.483	2	6.212	2	2.925	2	1.101
1	100.21	1	76.74	1	57.52	1	41.77	1	29.02	1	18.97	1	11.419	1	6.170	1	2.901	1	1.087
0	99.95	0	76.53	0	57.35	0	41.63	0	28.90	0	18.88	0	11.356	0	6.128	0	2.877	0	1.074
9.89	99.69	8.89	76.32	7.89	57.17	6.89	41.49	5.89	28.79	4.89	18.79	3.89	11.292	2.89	6.086	1.89	2.853	0.89	1.060
8	99.44	8	76.11	8	57.00	8	41.35	8	28.68	8	18.70	8	11.229	8	6.045	8	2.829	8	1.047
7	99.18	7	75.90	7	56.83	7	41.21	7	28.57	7	18.62	7	11.167	7	6.003	7	2.805	7	1.034
6	98.92	6	75.69	6	56.66	6	41.07	6	28.45	6	18.53	6	11.104	6	5.962	6	2.781	6	1.021
5	98.66	5	75.48	5	56.48	5	40.93	5	28.34	5	18.44	5	11.042	5	5.921	5	2.758	5	1.008
9.84	98.41	8.84	75.27	7.84	56.31	6.84	40.79	5.84	28.23	4.84	18.36	3.84	10.980	2.84	5.881	1.84	2.735	0.84	0.995
3	98.15	3	75.06	3	56.14	3	40.65	3	28.12	3	18.27	3	10.918	3	5.841	3	2.712	3	.982
2	97.90	2	74.85	2	55.97	2	40.51	2	28.01	2	18.19	2	10.856	2	5.800	2	2.688	2	.969
1	97.64	1	74.64	1	55.80	1	40.37	1	27.90	1	18.10	1	10.795	1	5.760	1	2.665	1	.956
0	97.39	0	74.44	0	55.63	0	40.23	0	27.78	0	18.02	0	10.734	0	5.720	0	2.642	0	.943
9.79	97.14	8.79	74.23	7.79	55.46	6.79	40.09	5.79	27.67	4.79	17.93	3.79	10.673	2.79	5.680	1.79	2.620	0.79	.931
8	96.88	8	74.02	8	55.29	8	39.95	8	27.56	8	17.85	8	10.612	8	5.641	8	2.598	8	.918
7	96.63	7	73.82	7	55.12	7	39.82	7	27.45	7	17.76	7	10.551	7	5.602	7	2.575	7	.906
6	96.38	6	73.61	6	54.95	6	39.68	6	27.34	6	17.68	6	10.491	6	5.563	6	2.553	6	.893
5	96.13	5	73.40	5	54.78	5	39.54	5	27.23	5	17.60	5	10.431	5	5.524	5	2.531	5	.881
9.74	95.88	8.74	73.20	7.74	54.62	6.74	39.40	5.74	27.12	4.74	17.51	3.74	10.371	2.74	5.485	1.74	2.509	0.74	.868
3	95.63	3	72.99	3	54.45	3	39.27	3	27.02	3	17.43	3	10.311	3	5.447	3	2.487	3	.856
2	95.38	2	72.79	2	54.28	2	39.13	2	26.91	2	17.34	2	10.252	2	5.408	2	2.465	2	.844
1	95.13	1	72.58	1	54.11	1	39.00	1	26.80	1	17.26	1	10.193	1	5.370	1	2.443	1	.832
0	94.88	0	72.38	0	53.94	0	38.86	0	26.69	0	17.18	0	10.134	0	5.332	0	2.422	0	.819
9.69	94.63	8.69	72.18	7.69	53.78	6.69	38.72	5.69	26.58	4.69	17.10	3.69	10.075	2.69	5.295	1.69	2.401	0.69	.807
8	94.38	8	71.98	8	53.61	8	38.59	8	26.48	8	17.02	8	10.017	8	5.257	8	2.380	8	.795
7	94.14	7	71.78	7	53.45	7	38.45	7	26.37	7	16.93	7	9.959	7	5.220	7	2.359	7	.783
6	93.89	6	71.57	6	53.28	6	38.32	6	26.26	6	16.85	6	9.901	6	5.183	6	2.338	6	.771
5	93.64	5	71.37	5	53.12	5	38.18	5	26.15	5	16.77	5	9.843	5	5.146	5	2.317	5	.759
9.64	93.40	8.64	71.17	7.64	52.95	6.64	38.05	5.64	26.05	4.64	16.69	3.64	9.785	2.64	5.109	1.64	2.296	0.64	.747
3	93.15	3	70.97	3	52.79	3	37.92	3	25.94	3	16.61	3	9.728	3	5.072	3	2.276	3	.735
2	92.91	2	70.77	2	52.62	2	37.78	2	25.84	2	16.53	2	9.671	2	5.036	2	2.256	2	.723
1	92.66	.1	70.57	1	52.46	1	37.65	1	25.73	1	16.45	1	9.614	1	5.000	1	2.236	1	.711
0	92.42	0	70.37	0	52.30	0	37.52	0	25.62	0	16.37	0	9.557	0	4.964	0	2.216	0	.699
9.59	92.18	8.59	70.17	7.59	52.13	6.59	37.38	5.59	25.52	4.59	16.29	3.59	9.501	2.59	4.928	1.59	2.196	0.59	.687
8	91.93	8	69.97	8	51.97	8	37.25	8	25.41	8	16.21	8	9.445	8	4.892	8	2.176	8	.675
7	91.69	7	69.78	7	51.81	7	37.12	7	25.31	7	16.13	7	9.389	7	4.857	7	2.156	7	.663
6	91.45	6	69.58	6	51.64	6	36.99	6	25.20	6	16.05	6	9.333	6	4.822	6	2.136		.651
5	91.21	5	69.38	5	51.48	5	36.86	5	25.10	5	15.97	5	9.277	5	4.787	5	2.116		.640
9.54	90.97	8.54	69.18	7.54	51.32	6.54	36.72	5.54	25.00	4.54	15.89	3.54	9.222	2.54	4.752	1.54	2.097	0	.628
3	90.73	3	68.99	3	51.16	3	36.59	3	24.89	3	15.81	3	9.167	3	4.717	3	2.078	5	.617
2	90.49	2	68.79	2	51.00	2	36.46	2	24.79	2	15.74	2	9.112	2	4.682	2	2.059	2	.605
1	90.25	1	68.59	1	50.84	1	36.33	1	24.69	1	15.66	1	9.058	1	4.648	1	2.040	1	.593
0	90.01	0	68.40	0	50.68	0	36.20	0	24.58	0	15.57	0	9.003	0	4.614	0	2.021	0	.581
9.49	89.77	8.49	68.20	7.49	50.52	6.49	36.07	5.49	24.48	4.49	15.49	3.49	8.949	2.49	4.580	1.49	2.002	0.49	.570
8	89.53	8	68.01	8	50.36	8	35.94	8	24.38	8	15.42	8	8.895	8	4.546	8	1.983	8	.559
7	89.30	7	67.81	7	50.20	7	35.81	7	24.28	7	15.34	7	8.841	7	4.512	7	1.965	7	.547
6	89.06	6	67.62	6	50.04	6	35.68	6	24.17	6	15.26	6	8.787	6	4.479	6	1.947	6	.535
5	88.82	5	67.43	5	49.88	5	35.56	5	24.07	5	15.18	5	8.734	5	4.446	5	1.929	5	.524
9.41	88.59	8.44	67.23	7.44	49.72	6.44	35.43	5.44	23.97	4.44	15.11	3.44	8.681	2.44	4.413	1.44	1.910	0.44	.513
3	88.35	3	67.04	3	49.56	3	35.30	3	23.87	3	15.03	3	8.628	3	4.380	3	1.892	3	.501
2	88.12	2	66.85	2	49.41	2	35.17	2	23.77	2	14.96	2	8.575	2	4.347	2	1.874	2	.489
1	87.88	1	66.66	1	49.25	1	35.04	1	23.67	1	14.88	1	8.523	1	4.314	1	1.856	1	.478
0	87.65	0	66.46	0	49.09	0	34.92	0	23.57	0	14.81	0	8.471	0	4.282	0	1.838	0	.467
9.39	87.41	8.39	66.27	7.39	48.93	6.39	34.79	5.39	23.47	4.39	14.73	3.39	8.419	2.39	4.250	1.39	1.821	0.39	.455
8	87.18	8	66.08	8	48.78	8	34.66	8	23.37	8	14.66	8	8.367	8	4.218	8	1.803	8	.444
7	86.95	7	65.89	7	48.62	7	34.54	7	23.27	7	14.58	7	8.316	7	4.186	7	1.786	7	.432
6	86.72	6	65.70	6	48.47	6	34.41	6	23.17	6	14.51	6	8.264	6	4.154	6	1.769	6	.421
5	86.48	5	65.51	5	48.31	5	34.28	5	23.07	5	14.43	5	8.213	5	4.123	5	1.752	5	.409
9.34	86.25	8.34	65.32	7.34	48.16	6.34	34.16	5.34	22.97	4.34	14.36	3.34	8.162	2.34	4.092	1.34	1.735	0.34	.398
3	86.02	3	65.13	3	48.00	3	34.03	3	22.87	3	14.28	3	8.111	3	4.060	3	1.718	3	.386
2	85.79	2	64.94	2	47.85	2	33.91	2	22.78	2	14.21	2	8.060	2	4.029	2	1.701	2	.375
1	85.56	1	64.76	1	47.69	1	33.78	1	22.68	1	14.14	1	8.010	1	3.998	1	1.684	1	.363
0	85.33	0	64.57	0	47.54	0	33.66	0	22.58	0	14.07	0	7.960	0	3.968	0	1.667	0	.352
9.29	85.10	8.29	64.38	7.29	47.38	6.29	33.54	5.29	22.48	4.29	13.99	3.29	7.910	2.29	3.938	1.29	1.650	0.29	.341
8	84.88	8	64.19	8	47.23	8	33.41	8	22.38	8	13.92	8	7.860	8	3.907	8	1.634	8	.329
7	84.65	7	64.01	7	47.08	7	33.29	7	22.29	7	13.85	7	7.811	7	3.877	7	1.618	7	.318
6	84.42	6	63.82	6	46.92	6	33.16	6	22.19	6	13.78	6	7.762	6	3.847	6	1.601	6	.306
5	84.19	5	63.63	5	46.77	5	33.04	5	22.09	5	13.70	5	7.713	5	3.817	5	1.585	5	.295
9.24	83.97	8.24	63.45	7.24	46.62	6.24	32.92	5.24	22.00	4.24	13.63	3.24	7.664	2.24	3.787	1.24	1.569	0.24	.283
3	83.74	3	63.26	3	46.47	3	32.80	3	21.90	3	13.56	3	7.615	3	3.758	3	1.553	3	.272
2	83.52	2	63.08	2	46.32	2	32.67	2	21.81	2	13.49	2	7.567	2	3.729	2	1.537	2	.260
1	83.29	1	62.89	1	46.17	1	32.55	1	21.71	1	13.42	1	7.519	1	3.700	1	1.521	1	.248
0	83.07	0	62.71	0	46.02	0	32.43	0	21.62	0	13.35	0	7.471	0	3.671	0	1.506	0	.237
9.19	82.84	8.19	62.52	7.19	45.87	6.19	32.31	5.19	21.52	4.19	13.28	3.19	7.423	2.19	3.642	1.19	1.490	0.19	.225
8	82.62	8	62.34	8	45.72	8	32.19	8	21.43	8	13.21	8	7.375	8	3.613	8	1.475	8	.214
7	82.39	7	62.16	7	45.57	7	32.07	7	21.33	7	13.14	7	7.328	7	3.585	7	1.459	7	.202
6	82.17	6	61.98	6	45.42	6	31.95	6	21.24	6	13.07	6	7.281	6	3.557	6	1.444	6	.191
5	81.95	5	61.79	5	45.27	5	31.83	5	21.14	5	13.00	5	7.234	5	3.529	5	1.429	5	.179
9.14	81.73	8.14	61.61	7.14	45.12	6.14	31.71	5.14	21.05	4.14	12.93	3.14	7.187	2.14	3.501	1.14	1.413	0.14	.167
3	81.50	3	61.43	3	44.97	3	31.59	3	20.96	3	12.86	3	7.140	3	3.473	3	1.398	3	.155
2	81.28	2	61.25	2	44.82	2	31.47	2	20.86	2	12.80	2	7.094	2	3.445	2	1.383	2	.143
1	81.06	1	61.07	1	44.67	1	31.35	1	20.77	1	12.73	1	7.048	1	3.418	1	1.368	1	.131
0	80.84	0	60.88	0	44.52	0	31.23	0	20.68	0	12.66	0	7.002	0	3.391	0	1.354	0	.120
9.09	80.62	8.09	60.70	7.09	44.38	6.09	31.11	5.09	20.59	4.09	12.59	3.09	6.956	2.09	3.364	1.09	1.339	0.09	.108
8	80.40	8	60.52	8	44.23	8	30.99	8	20.49	8	12.52	8	6.911	8	3.337	8	1.324	8	.096
7	80.18	7	60.35	7	44.08	7	30.87	7	20.40	7	12.46	7	6.866	7	3.310	7	1.310	7	.084
6	79.97	6	60.17	6	43.94	6	30.75	6	20.31	6	12.39	6	6.821	6	3.283	6	1.295	6	.073
5	79.75	5	59.99	5	43.79	5	30.64	5	20.22	5	12.32	5	6.776	5	3.256	5	1.281	5	.061
9.04	79.53	8.01	59.81	7.04	43.64	6.04	30.52	5.04	20.13	4.04	12.26	3.04	6.731	2.04	3.230	1.04	1.267	0.04	.049
3	79.31	3	59.63	3	43.50	3	30.40	3	20.04	3	12.19	3	6.687	3	3.204	3	1.253	3	.036
2	79.10	2	59.45	2	43.35	2	30.28	2	19.95	2	12.12	2	6.643	2	3.178	2	1.238	2	.024
1	78.88	1	59.28	1	43.21	1	30.17	1	19.86	1	12.06	1	6.599	1	3.152	1	1.224	1	.012
0	78.66	0	59.10	0	43.06	0	30.05	0	19.77	0	12.00	0	6.555	0	3.126	0	1.210	0	.000

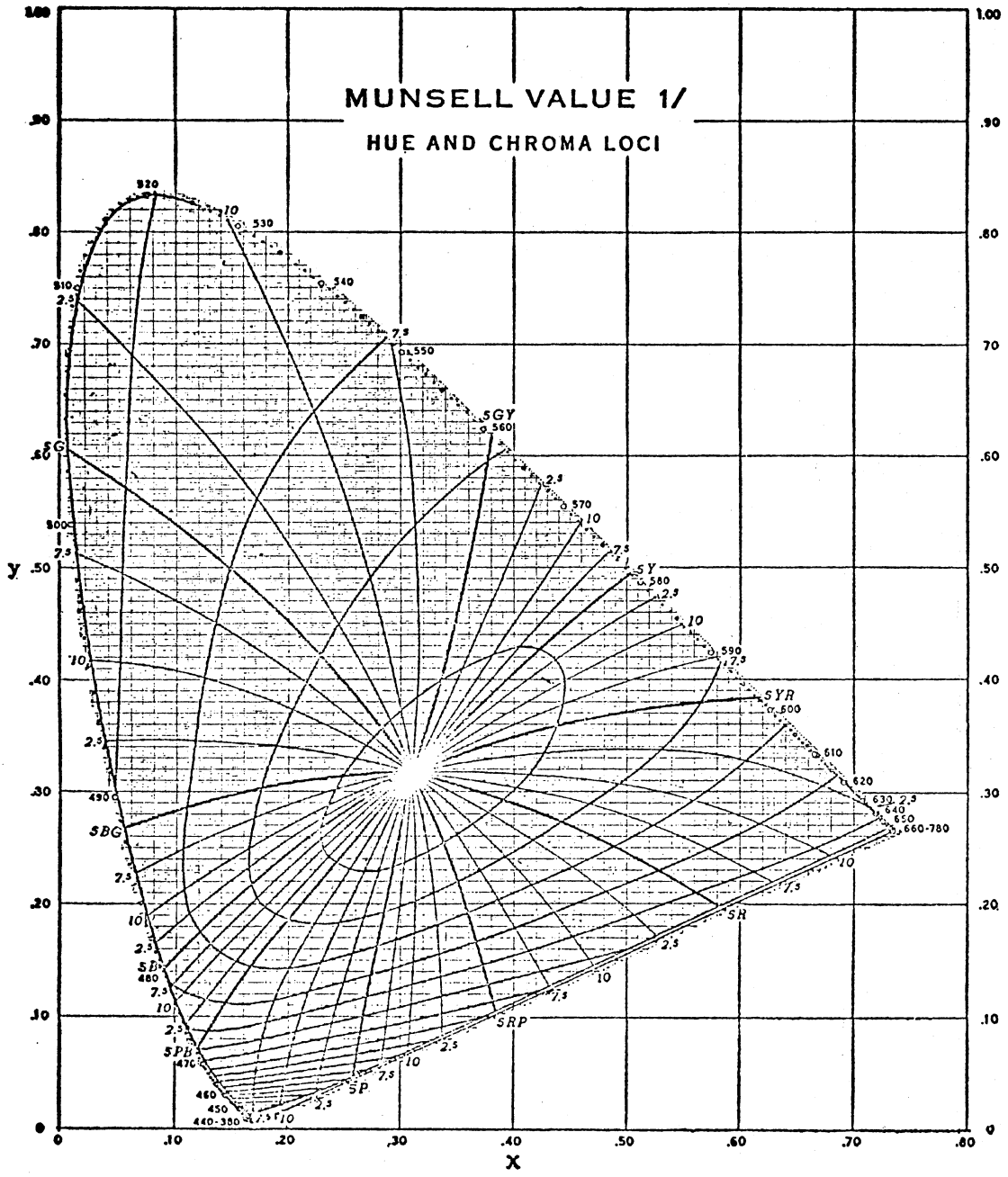


Figure 26. Loci of constant-hue and constant-chroma in I.C.I. (x,y) coordinates at value 1/ for I.C.I. illuminant C. (13).

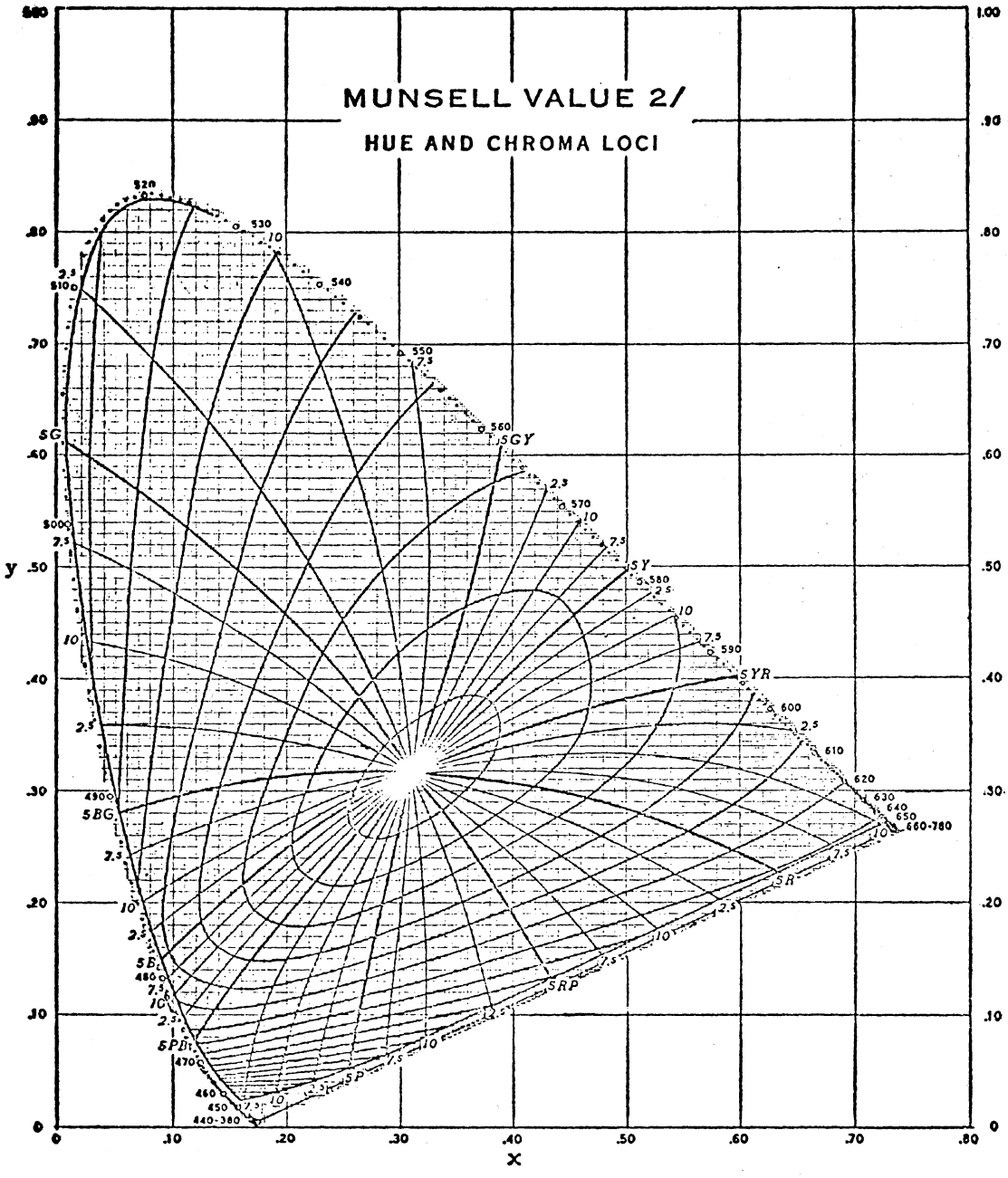


Figure 27. Loci of constant-hue and constant-chroma in I.C.I. (x,y) coordinates at value 2/ for I.C.I. illuminant C (3).

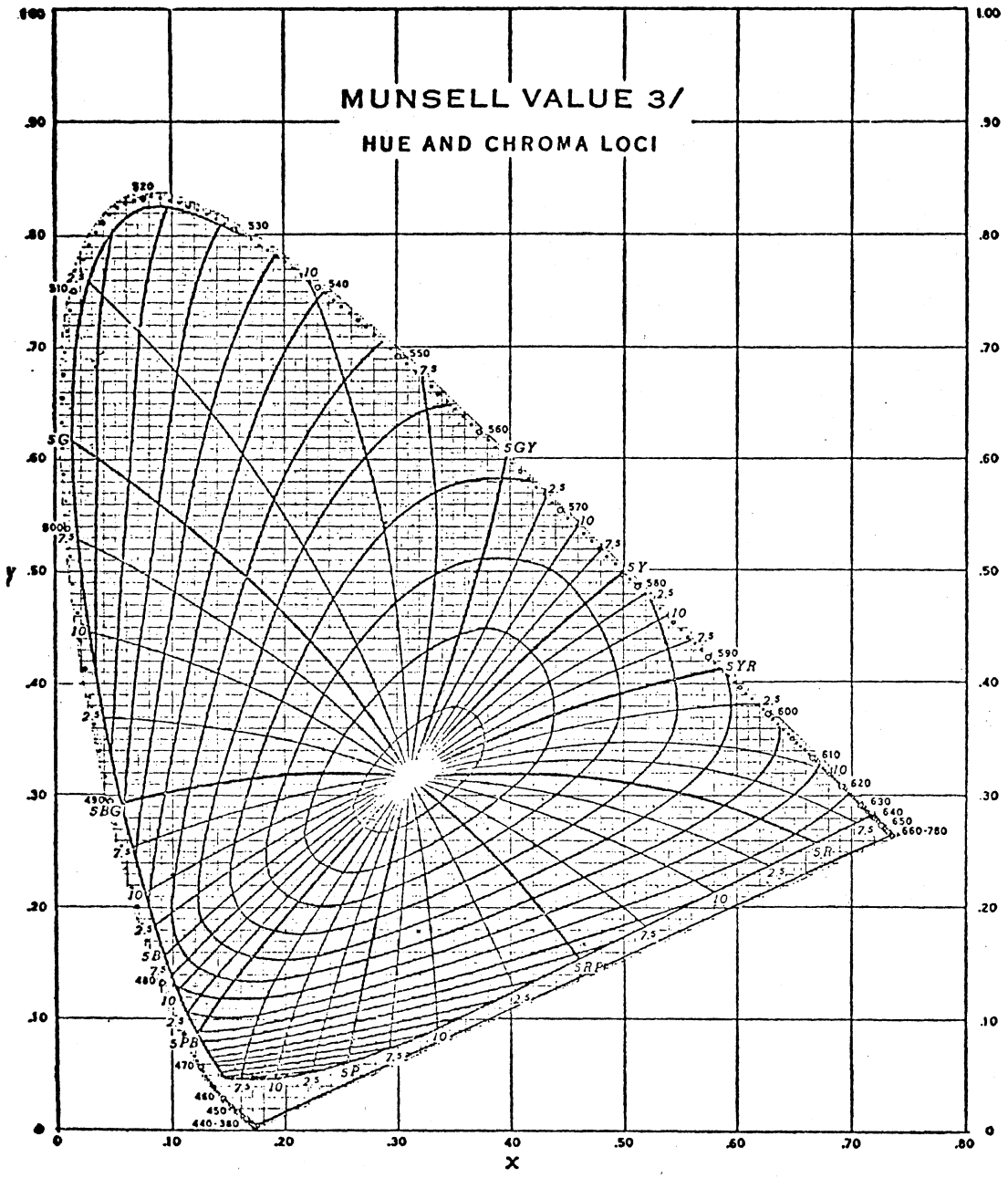


Figure 28. Loci of constant-hue and constant-chroma in I.C.I. (x,y) coordinates at value 3/ for I.C.I. illuminant C (13).

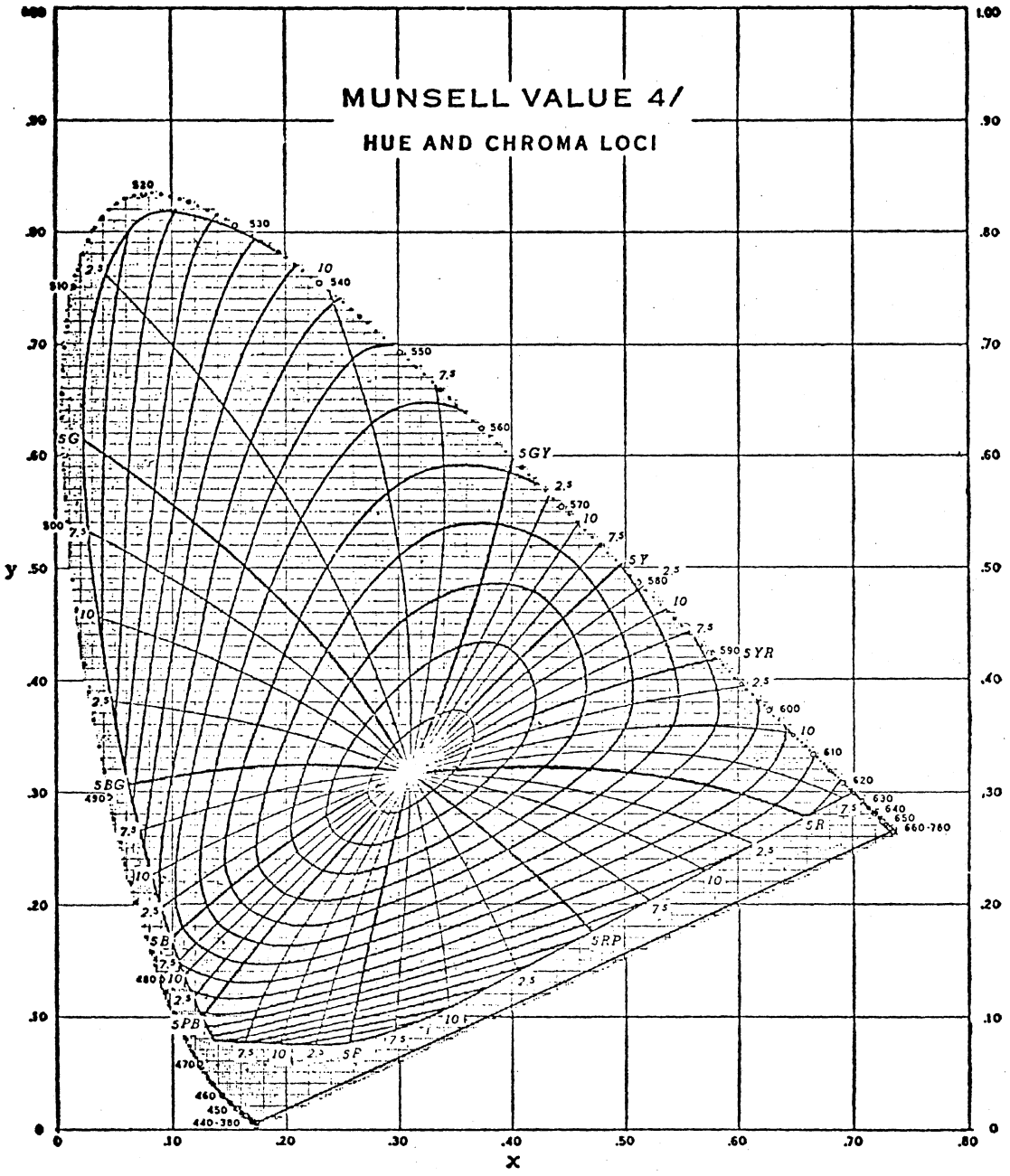


Figure 29. Loci of constant-hue and constant-chroma in I.C.I. (x,y) coordinates at value 4/ for I.C.I. illuminant C (13).

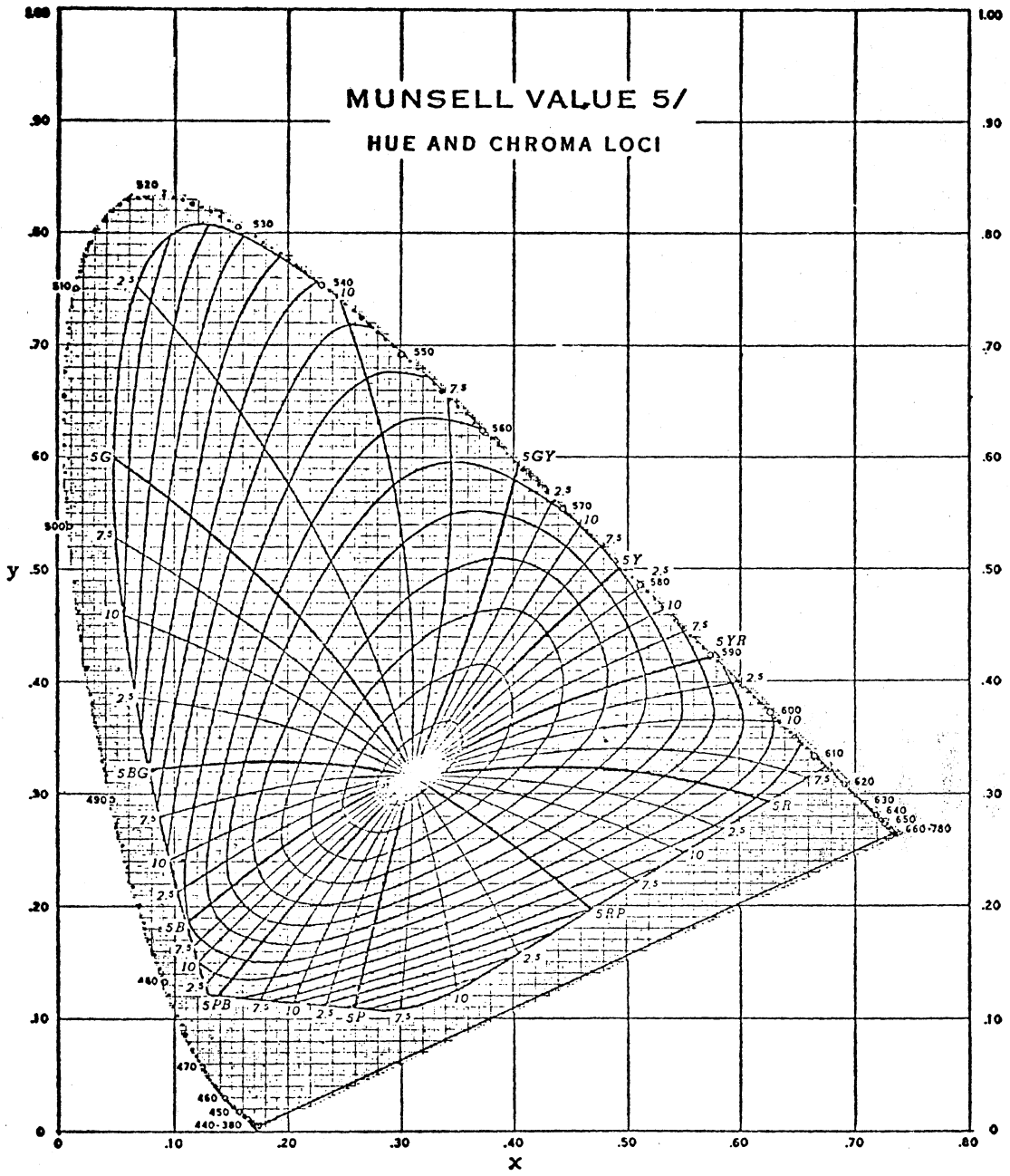


Figure 30. Loci of constant-hue and constant-chroma in I.C.I. (x,y) coordinates at value 5/ for I.C.I. illuminant C (13).

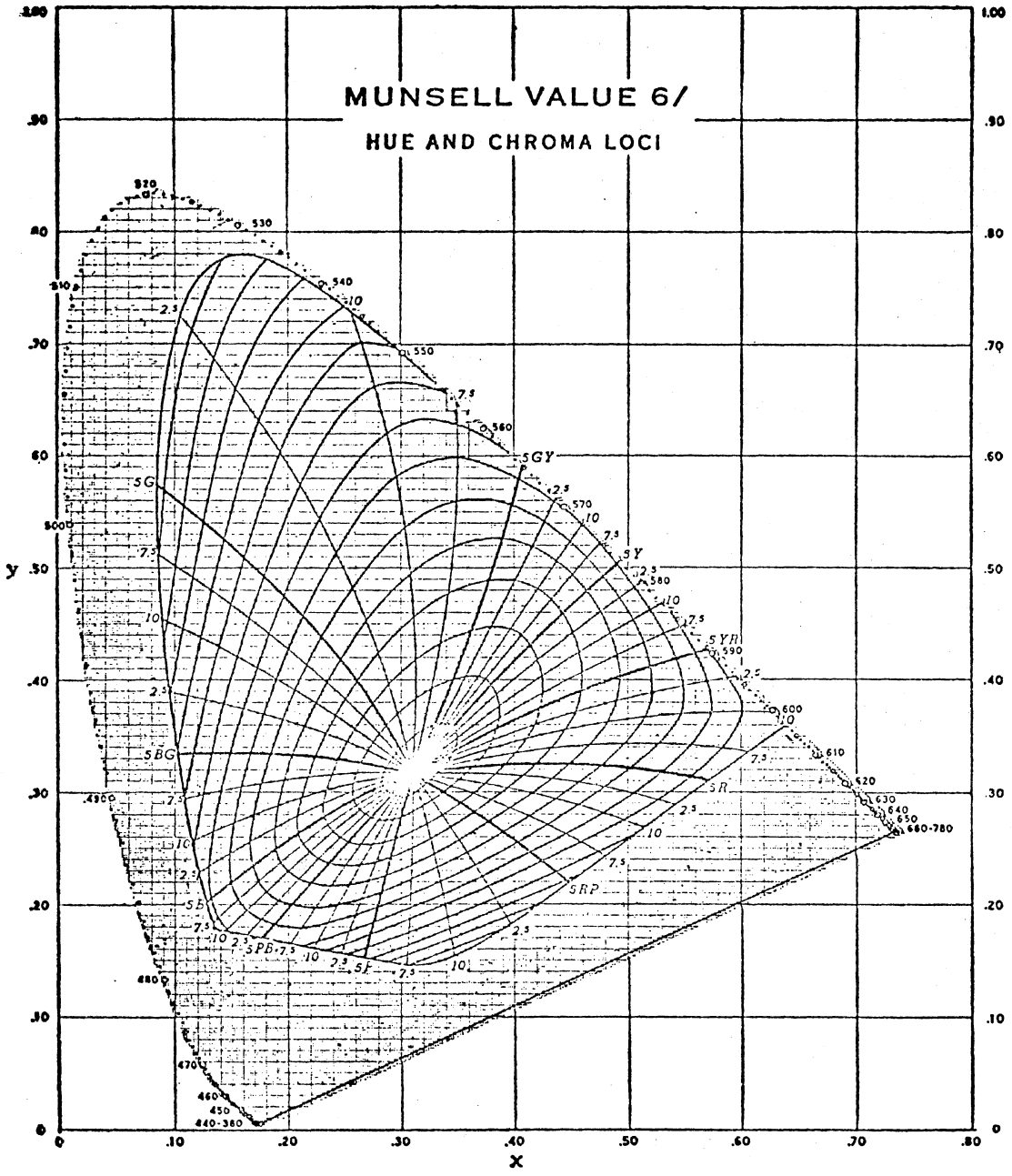


Figure 31. Loci of constant-hue and constant-chroma in I.C.I. (x,y) coordinates at value 6/ for I.C.I. illuminant C (13).

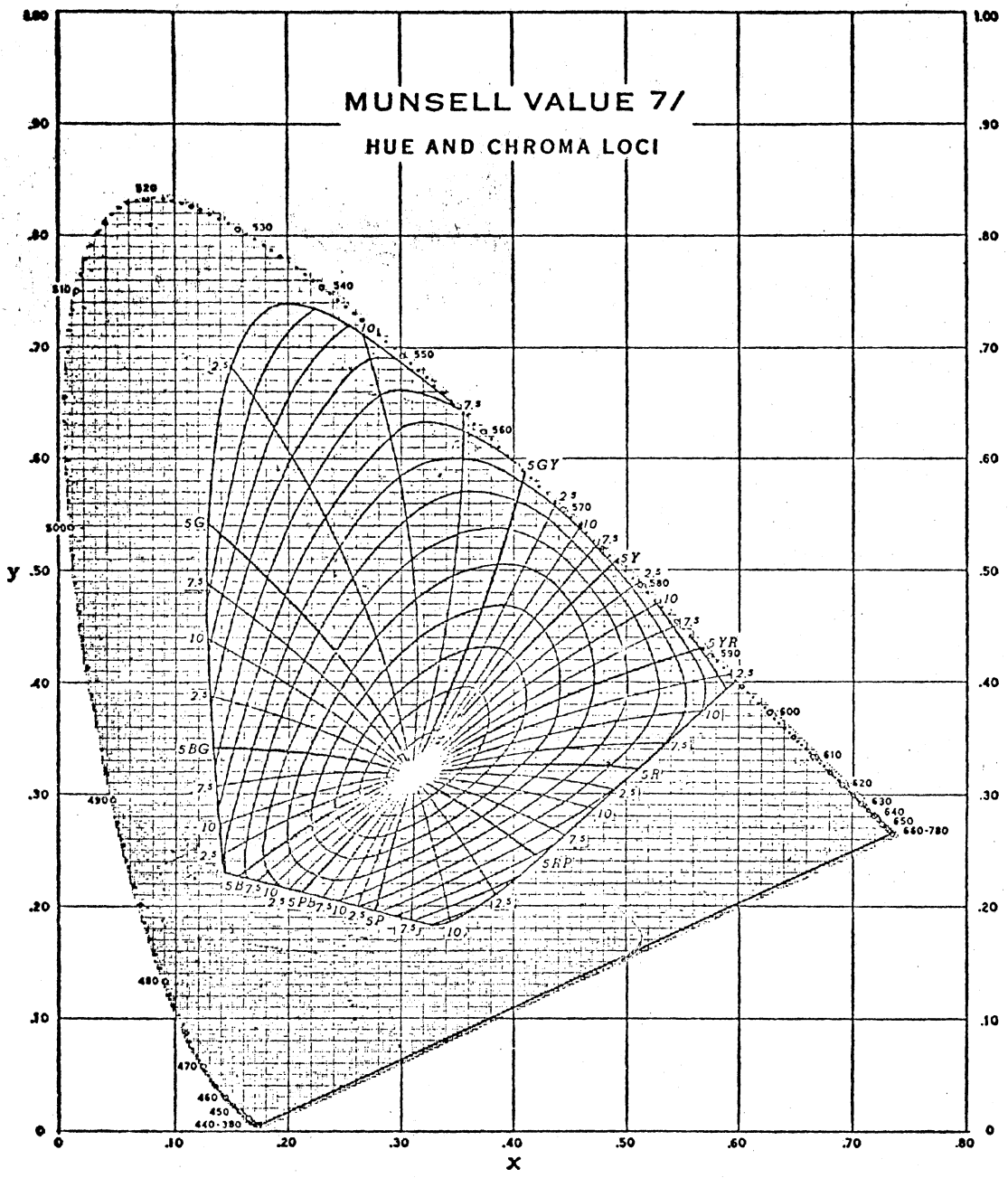


Figure 32. Loci of constant-hue and constant-chroma in I.C.I. (x,y) coordinates at value 7/ for I.C.I. illuminant C (13).

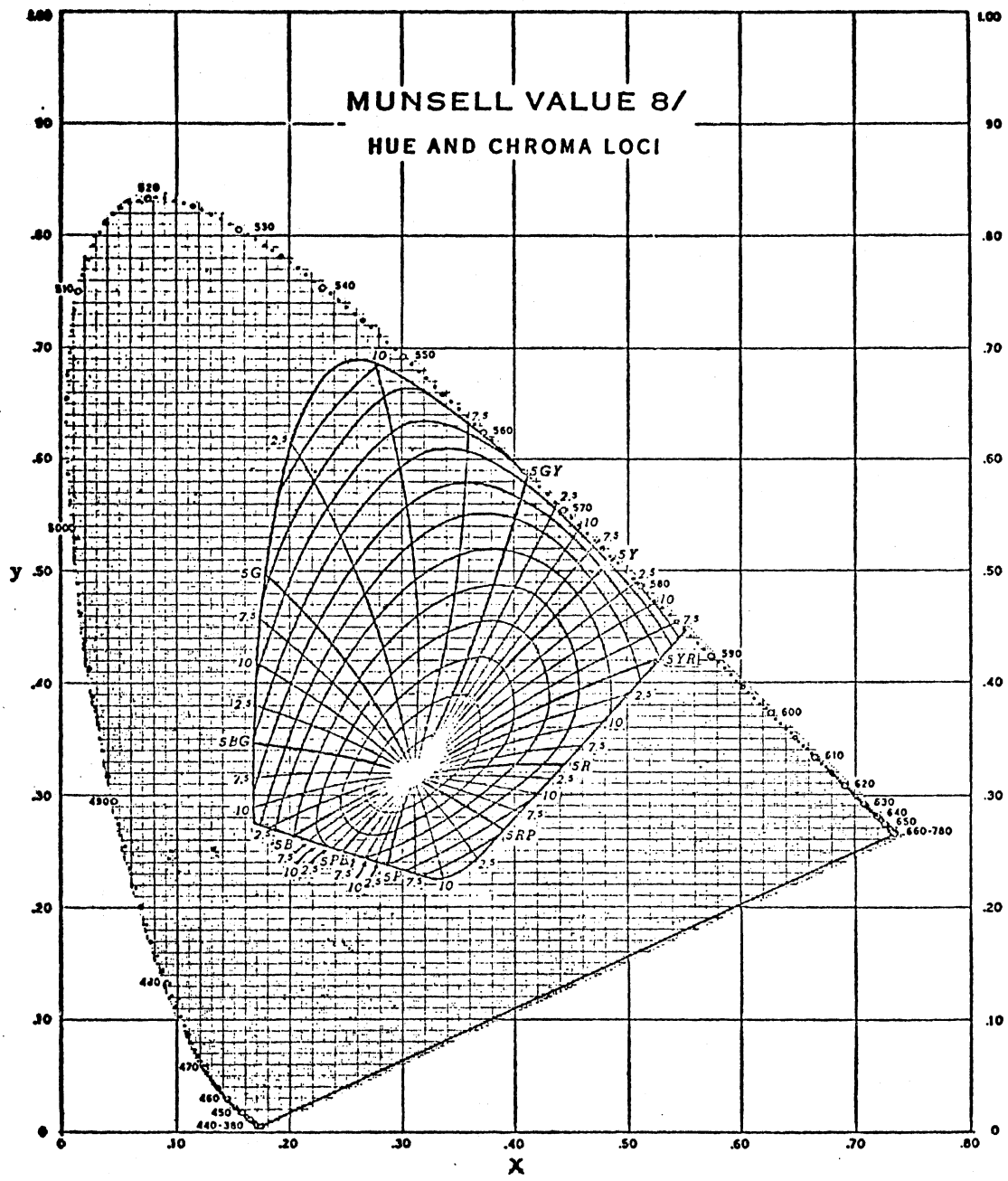


Figure 33. Loci of constant-hue and constant-chroma in I.C.I. (x,y) coordinates at value 8/ for I.C.I. illuminant C (13).

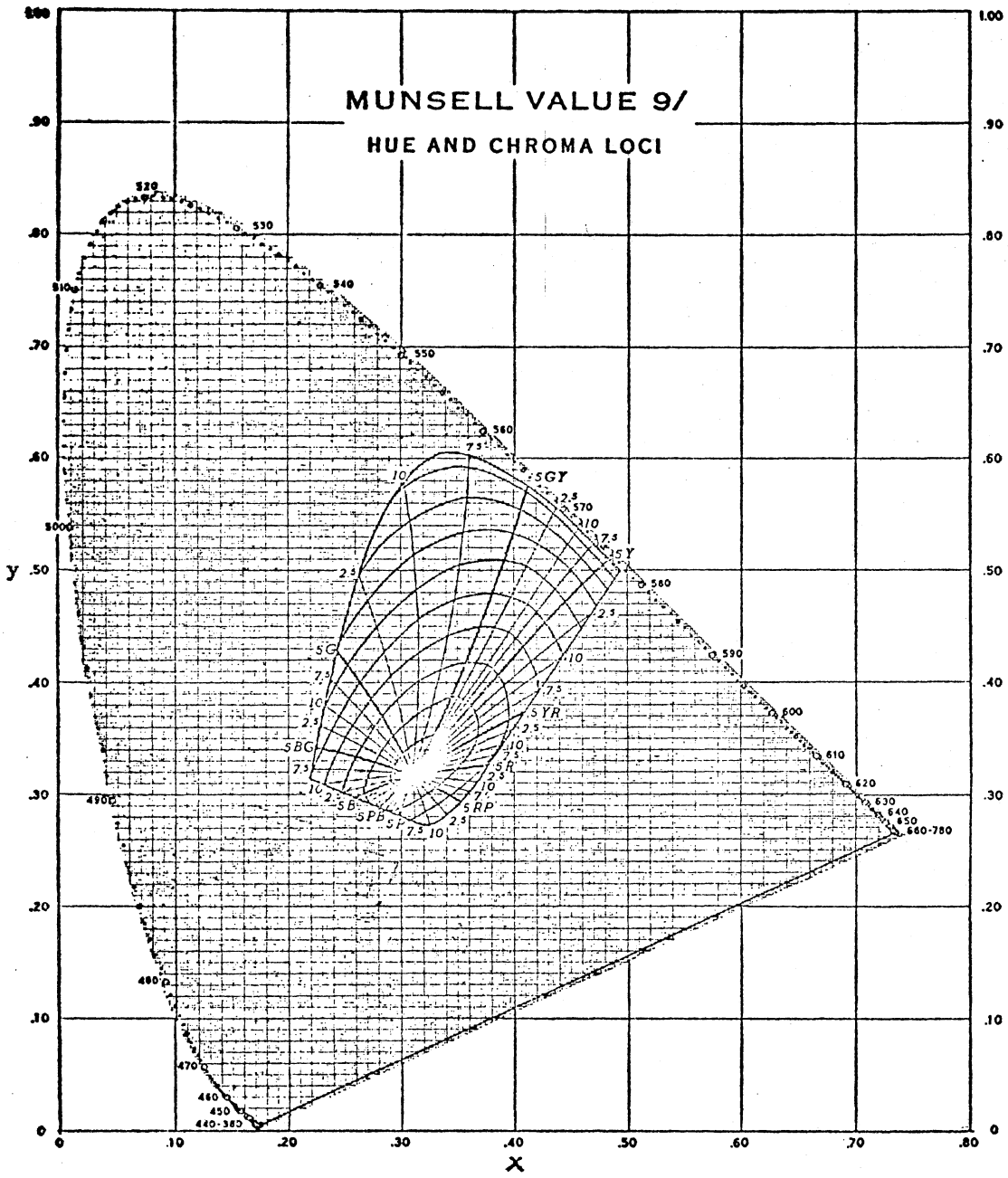


Figure 34. Loci of constant-hue and constant-chroma in I.C.I. (x,y) coordinates at value 9/ for I.C.I. illuminant C (13).

To find the hue, the chromaticity co-ordinates (x_c,y_c) of the sample are plotted on the value chart V calculated from Y_r. A line is then drawn from point (x_w,y_w) through (x_c,y_c) outwards to intersect the spectrum locus. Figure 35 illustrates this procedure. The point of intersection of the straight line and the spectrum will give the dominant wavelength of the sample. In Figure 35 (x_d,y_d) has a dominant wavelength λ_d = 505nm (8).

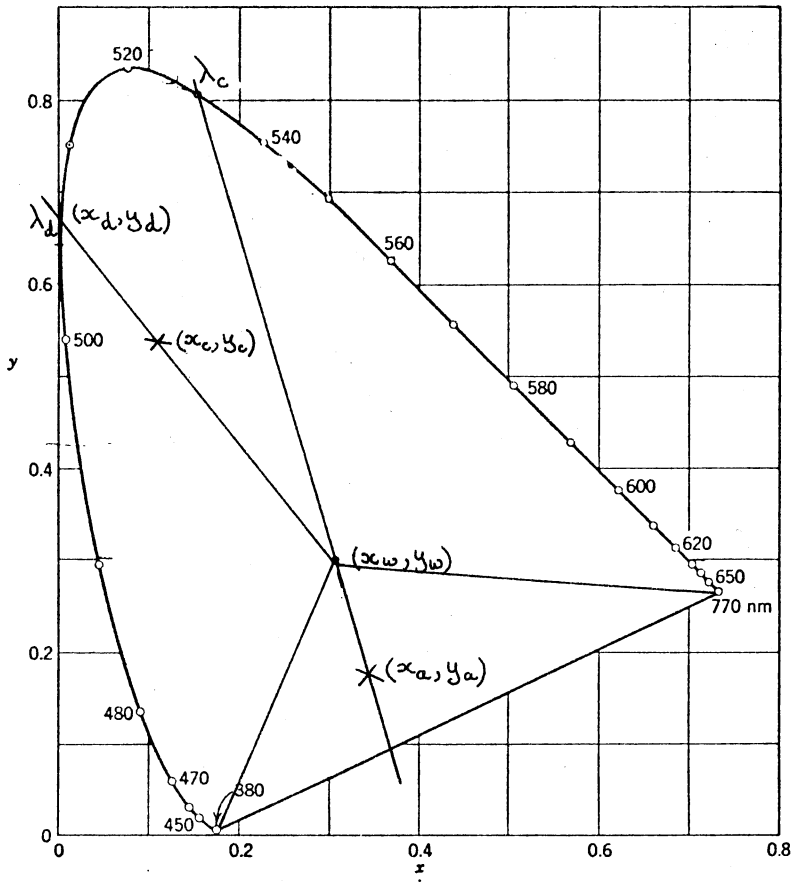


Figure 35. Illustration of methods of determining dominant wavelength λ_d and complementary wave length λ_c.

If the point representing (x_w,y_w) lies between the points representing the color and the intersection with the spectrum locus, then the wave-length corresponding to the intersection is the complementary wavelength λ_c of the color. The complementary wavelength of (x_a,y_a) represented in Figure 35 is λ_c = 530 nm. In lists of dominant wavelengths such a sample is commonly designated 530_c. Colors having chromaticites located within the triangle formed by the point representing (x_w,y_w) and the extremities of the spectrum locus have no dominant wavelengths. Colors having such chromaticities are called non-spectral colors and are specified in terms of their complementary wavelengths. Colors having chromaticites represented within the remainder of the domain of real colors are called spectral colors. Colors represented by points on the spectrum locus are called spectrum colors. Once the dominant wavelength is determined from the chromaticity chart, the hue of the sample is known also. The lines which radiate outward, on the chromaticity charts, from the illuminant C point (x_w,y_w) are lines of constant hue. If the sample point (x_c,y_c) does not fall on any of these lines then interpolation between the lines has to be used to determine the hue.

The closed curves which encircle the illuminant C point (x_w,y_w) are loci of constant chroma for that particular Munsell value. If the sample point (x_c,y_c) falls on one of these loci, then that is the corresponding chroma for that sample. If the sample point (x_c,y_c) falls between two loci then once again interpolation has to be used. The locus closest to the illuminant C point (x_w,y_w) represents chroma (2). The next locus out represents chroma (4) and so on.

It should be noted, from the value charts, that for the light end

of the Munsell values scale 9 through 5 the contours of constant Munsell chroma (2,4,6, and so on) increase in size only slightly for each darker step in value scale. The chief difference in the Munsell chroma for the light end of the value scale (V=10) is that they cover a much larger fraction of the chromaticity diagram at 5 value than they do a 9 value. For Munsell values between 0 and 1 interpolation is unreliable because the locations of the Munsell chroma loci for Munsell value 0 are entirely off the chromaticity diagram.

Figure 36 illustrates a master hue chart in I.C.I. (x,y) chromaticity coordinates showing the recommended loci of constant hue for 20 standard Munsell hues at value levels 1 through 9.

While it has been the objective of this chapter to show how the Munsell system was developed and implemented, it should be observed that a great deal of time and effort is needed to obtain the Munsell color representation of a sample, especially if interpolation is needed. To help alleviate some of this trouble and improve the accuracy of the task a computer program was developed so that the visual interpolation by charts is not needed. Chapter V describes the development and implementation of this program.

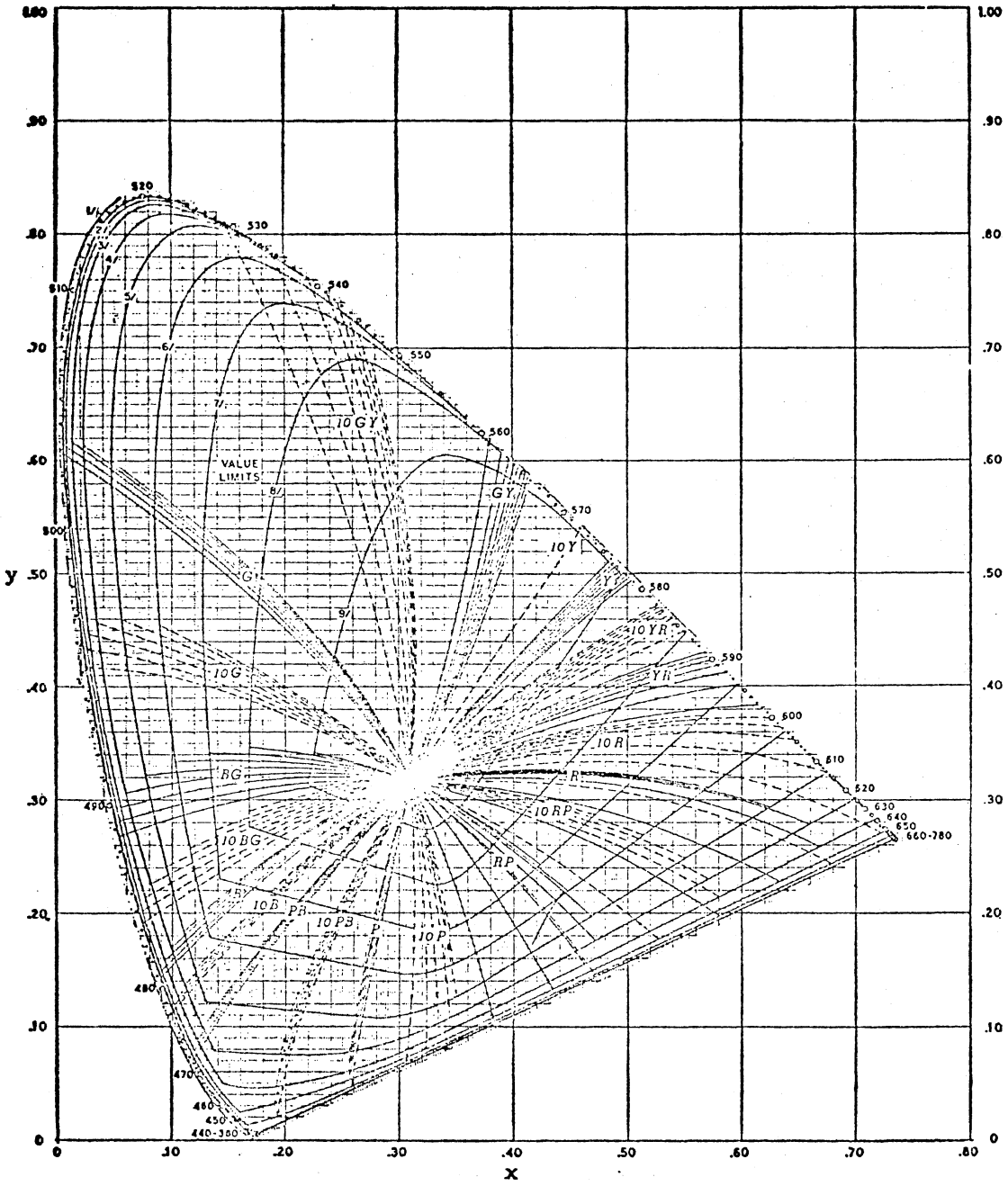


Figure 36. Master hue chart in I.C.I. (x,y) coordinates showing the recommended loci of constant hue for 20 standard Munsell hues at value levels 1/ through 9/. The families of hue lines indicate the change in dominant wavelength that is necessary to keep hue constant at various value and chroma levels. The closed lines indicate the theoretical (MacAdam) limit beyond which nonfluorescent surface colors are not possible for the different value levels 1/ to 9/. The center closed line represents the limit colors available at value 9/; the limit colors at 1/ extend almost to the spectrum locus. The hue lines extend only to the theoretical limit for the value they represent.(13)

CHAPTER V

PROGRAM DEVELOPMENT

Any three lights may be used as primaries in a system of tristimulus color specification, provided only that no one of them is equivalent to a combination of the other two. Tristimulus specifications X, Y, Z expressed relative to one set of primaries may be transformed into specifications, R(red), G(green), and B(blue) relative to any other set of primaries by transformation equations of the form:

$$\begin{aligned} R &= K_1X + K_2Y + K_3Z \\ G &= K_4X + K_5Y + K_6Z \\ B &= K_7X + K_8Y + K_9Z \end{aligned} \tag{5.1}$$

The constants K_1 to K_9, may take on any arbitrary values, positive, negative, or zero, provided they are not such as to make one of the new primaries identical to a combination of the other two; that is provided that the determinant of the coefficient matrix of (5.1) is nonzero:

$$\begin{vmatrix} K_1 & K_2 & K_3 \\ K_4 & K_5 & K_6 \\ K_7 & K_8 & K_9 \end{vmatrix} \neq 0 \tag{5.2}$$

Since the exceptions that cause the determinant of the system to vanish are trivial, the choice of coordinate system is very unconstrained.

It is generally presumed that the initial responses of the eye are photochemcial in nature, and that there are three of them, each independent of the other two. If we knew what colors (imaginary or real) corresponded to each of these responses, we could evaluate the constants K_1 to K_9 in (5.1) and find the photo sensitive responses required for an observer.

Figure 37 is a schematic diagram of a filter-photocell colorimeter.

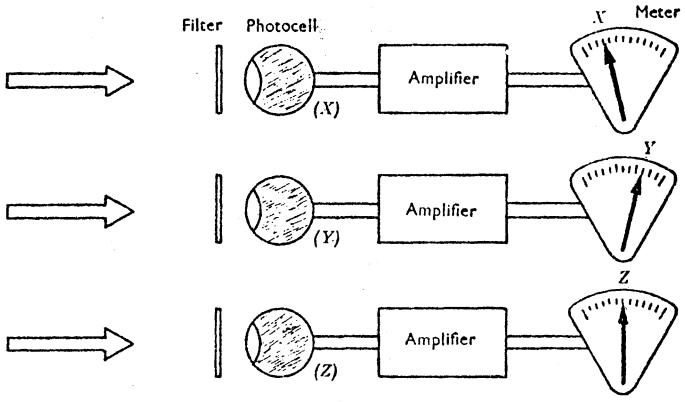


Figure 37. The Principle of the Photoelectric Tricolorimeter (10).

Radiant flux (reflected light) leaving the specimen passes through one of the three tristimulus filters (x), (y), (z) and falls onto a photocell causing it to give a response proportional to the corresponding tristimulus value of the specimen-source combination. Each tristimulus value filter is usually a combination of colored glass filters so chosen that the resultant spectral-transmittance function of the combination corrects the spectral-response function of the photocell to one of the CIE color-matching functions. If these three photocells could be adjusted, as by glass filters, so that their responses are proportional throughout the visible responses to some linear combination (as in 5.1), of the standard ICI distribution curves (see Figure 15) then they could be used to test whether any two light beams have the same color, and could be made to yield direct measurements of tristimulus values, X, Y and Z (9).

If A, G, and B represent the settings obtained for a specimen relative to those for a standard magnesium oxide surface with amber, green, and blue filters (the spectrometer uses an amber filter instead of a red filter) respectively, approximate tristimulus values, X, Y and Z may be found as (13)

$$\begin{aligned} X &\doteq 0.80A + 0.18B \\ Y &\doteq 1.00G \\ Z &\doteq 1.18B \end{aligned} \tag{5.3}$$

Approximate chromaticity coordinates, x,y, may then be found in the usual way:

$$\begin{aligned} x &= \frac{X}{(X + Y + Z)} \\ y &= \frac{Y}{(X + Y + Z)} \end{aligned} \tag{5.4}$$

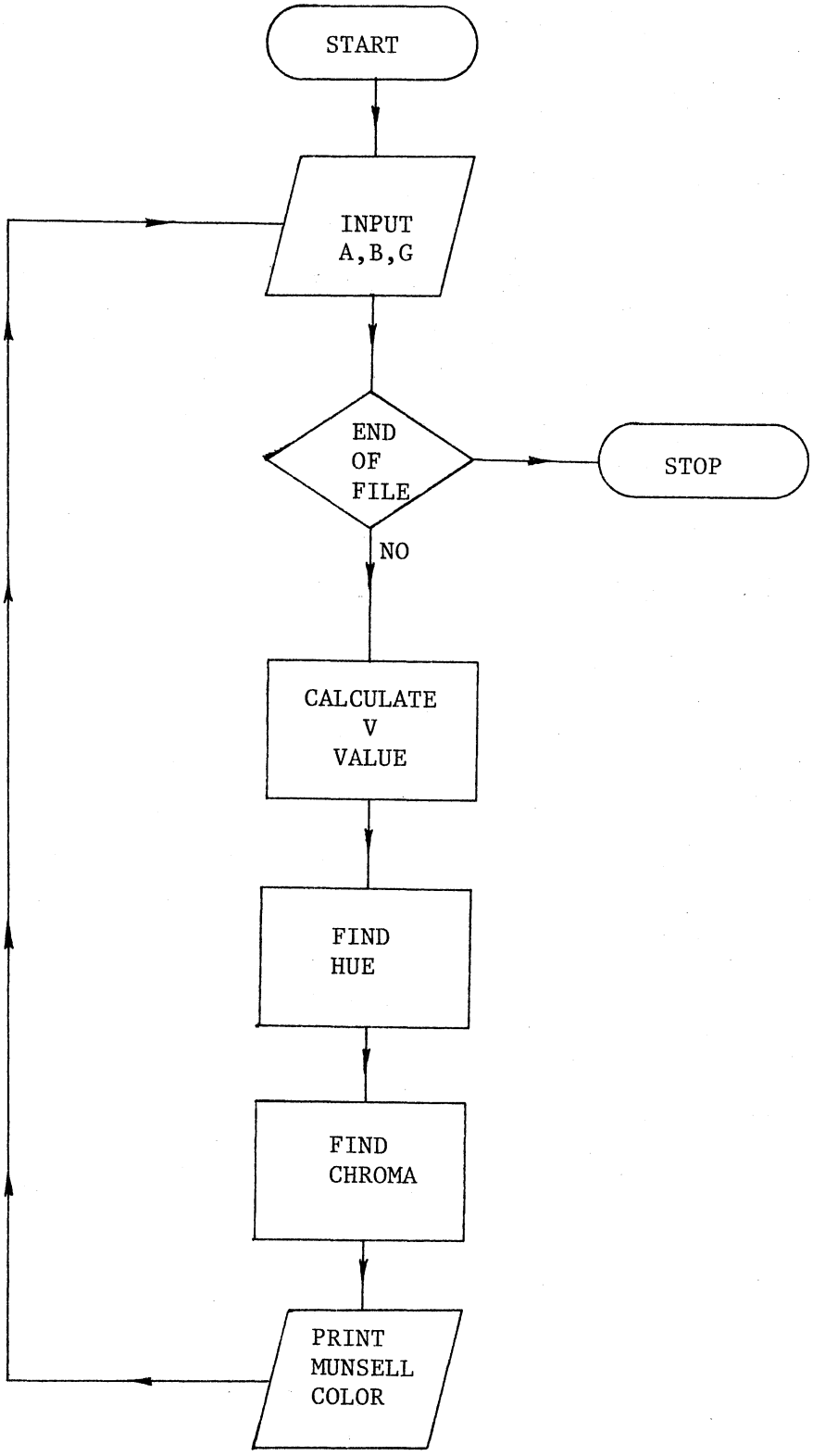


Figure 38. Basic flowchart for Munsell Color Program Development.

The Munsell color notation is then found by, using Y, x,y, and the charts of Chapter IV.

The computer program receives the spectrometer readings A,G, B and by a sequence of subroutine calls proceeds to produce the Munsell color for these three readings. Obviously, some operations have to be performed before others. The chroma and hue cannot be found until the value V has been calculated, the reason being that the V value designates which value charts are to be used with the x,y coordinates. With this in mind the flowchart of Figure 38 was drawn and the computer program developed and expanded from this foundation.

A subroutine was written to take a given set of A, G, B values and check to see if they are valid (between 0 and 100). Any errors detected are removed before the program continues. Appropriate error messages are printed and the erroneous data removed. The program will automatically loop to read in the next set of A, B, and B readings. If an end-of-file condition is raised, execution of the program is terminated.

The next step was to determine a way to calculate the V value from the input data, remembering from Chapters III and IV that the V value is found using tristimulus coordinates (x,y). In the case of the A, G and B data, this is just the numeric value of G (see 5.3). In the manual system, knowing Y, the corresponding V value was found by reference to Table III. At first the idea of reading Table III data into the program was considered. The V value could then be found by some search technique. This method would include: 1) storage for the 1001 data points of Table III; 2) development of a search procedure and maybe an interpolation procedure to calculate V: 3) increase in execution time, at the expense of the user. After short thought it was

decided to reject this primitive method.

The second approach was to fit a curve to the known data points of Table III. Then the equation of the curve could be coded into the program and a root finder could be used to find V. This approach was tried with various unconstrained minimization and least squares programs. Curves were found but the fits were very poor. This approach was dropped. Then, on close inspection of the data in Table III it was realized that the points were "too perfect". Obviously, they were not generated by experimental procedures, but by some predetermined equation.

The third approach was to try to find an article or book which would contain the required equation. This approach was successful. The equation was found (5.4).

$$Y_V = 1.2219V - 0.23111V^2 + 0.23951V^3 - 0.021009V^4 + 0.0008404V^5 \quad (5.4)$$

The derivation of this equation is explained in Chapter IV.

The equation could now be coded into the program. Now the problem of how to find V arose. If we had the value of V, clearly, substitution into the right hand side of (5.4) would produce Y very easily. Unfortunately, the Y value is known and the V value is to be produced. Obviously, some root finding technique would be needed. On looking at the curve (Figure 25) of (5.4) it was noted that the function is convex upward. This suggested Newton's method. Newton's process may be called analytic substitution, that is, for a function we could not handle we substitute another function on which we can perform the analytic operation. In Newton's iterative method for finding a zero of a function $y = f(x)$, we are given an approximate value X_1, and inplace of the curve we use the line

$$y - y_1 = m(x-x_1) \quad (5.5)$$

which is tangent to the curve at the point (x_1, y_1). The zero of the line is given by y = 0 or

$$x = x_1 - y_1/m \tag{5.6}$$

This new value of x is used as the next approximate value of the zero. The procedure then iterates, using each new approximation until the zero is found. Of course, an exact zero may not always be found because of contamination by roundoff noise. It is therefore necessary to terminate the iterations when the new approximation is within some delta of the zero.

With this idea in mind, Newton's method was coded as a subroutine with the initial $V(X_1)$ estimate as 5.0 and delta equal to 0.0000001. The subroutine was then tested with all 1001 Y values of Table III with the result that all the corresponding V values were reproduced exactly. It should also be noted that no resulting V value required more than 15 x_1 iterations. The form of equation (5.4) used in Newton's method was:

$$y = 1.2219V - 0.23111V^2 + 0.23951V^3 - 0.021009V^4 + 0.000840V^5 - Y_V \tag{5.7}$$

The Y_V value is now a constant term. The right hand side of (5.7) represents f(x) in y = f(x). Setting y = 0, we can use Newton's method, via the use of equations (5.4) and (5.6) to produce value V.

The next step in the development was to find the hue. In the manual method this was done by plotting the x,y coordinates on the corresponding value V chart. The hue was then found by connecting the x,y coordinates to the standard illuminant coordinates and extrapolating the line until it intersected the spectrum locus curve. This gives two pieces of information -- 1) the dominant wavelength of our object whose

TABLE IV

SPECTRUM LOCUS DATA

λ (nm)	$x(\lambda)$	$y(\lambda)$	$z(\lambda)$	λ (nm)	$x(\lambda)$	$y(\lambda)$	$z(\lambda)$
				575	0·4788	0·5202	0·0010
380	0·1741	0·0050	0·8209	580	0·5125	0·4866	0·0009
385	0·1740	0·0050	0·8210	585	0·5448	0·4544	0·0008
390	0·1738	0·0049	0·8213	590	0·5752	0·4242	0·0006
395	0·1736	0·0049	0·8215	595	0·6029	0·3965	0·0006
400	0·1733	0·0048	0·8219	600	0·6270	0·3725	0·0005
405	0·1730	0·0048	0·8222	605	0·6482	0·3514	0·0004
410	0·1726	0·0048	0·8226	610	0·6658	0·3340	0·0002
415	0·1721	0·0048	0·8231	615	0·6801	0·3197	0·0002
420	0·1714	0·0051	0·8235	620	0·6915	0·3083	0·0002
425	0·1703	0·0058	0·8239	625	0·7006	0·2993	0·0001
430	0·1689	0·0069	0·8242	630	0·7079	0·2920	0·0001
435	0·1669	0·0086	0·8245	635	0·7140	0·2859	0·0001
440	0·1644	0·0109	0·8247	640	0·7190	0·2809	0·0001
445	0·1611	0·0138	0·8251	645	0·7230	0·2770	0·0000
450	0·1566	0·0177	0·8257	650	0·7260	0·2740	0·0000
455	0·1510	0·0227	0·8263	655	0·7283	0·2717	0·0000
460	0·1440	0·0297	0·8263	660	0·7300	0·2700	0·0000
465	0·1355	0·0399	0·8246	665	0·7311	0·2689	0·0000
470	0·1241	0·0578	0·8181	670	0·7320	0·2680	0·0000
475	0·1096	0·0868	0·8036	675	0·7327	0·2673	0·0000
480	0·0913	0·1327	0·7760	680	0·7334	0·2666	0·0000
485	0·0687	0·2007	0·7306	685	0·7340	0·2660	0·0000
490	0·0454	0·2950	0·6596	690	0·7344	0·2656	0·0000
495	0·0235	0·4127	0·5638	695	0·7346	0·2654	0·0000
500	0·0082	0·5384	0·4534	700	0·7347	0·2653	0·0000
505	0·0039	0·6548	0·3413	705	0·7347	0·2653	0·0000
510	0·0139	0·7502	0·2359	710	0·7347	0·2653	0·0000
515	0·0389	0·8120	0·1491	715	0·7347	0·2653	0·0000
520	0·0743	0·8338	0·0919	720	0·7347	0·2653	0·0000
525	0·1142	0·8262	0·0596	725	0·7347	0·2653	0·0000
530	0·1547	0·8059	0·0394	730	0·7347	0·2653	0·0000
535	0·1929	0·7816	0·0255	735	0·7347	0·2653	0·0000
540	0·2296	0·7543	0·0161	740	0·7347	0·2653	0·0000
545	0·2658	0·7243	0·0099	745	0·7347	0·2653	0·0000
550	0·3016	0·6923	0·0061	750	0·7347	0·2653	0·0000
555	0·3373	0·6589	0·0038	755	0·7347	0·2653	0·0000
560	0·3731	0·6245	0·0024	760	0·7347	0·2653	0·0000
565	0·4087	0·5896	0·0017	765	0·7347	0·2653	0·0000
570	0·4441	0·5547	0·0012	770	0·7347	0·2653	0·0000
				775	0·7347	0·2653	0·0000
				780	0·7347	0·2653	0·0000

color is to be determined and 2) the hue of the object, found indirectly, via the dominant wavelength.

Finding the equation of the line is trivial and was done as follows.

Let (x_a, y_a) be the coordinates calculated from the A, G, and B values.

Let (x_w, y_w) be the coordinates of the standard illuminant used (in the case of the program this was assumed to be illuminant C.)

Using equation:

$$y - y_a = m(x - x_a)$$

$$y = mx - mx_a + y_a$$

$$y = mx + (y_a - mx_a)$$

$$y = mx + c \tag{5.8}$$

where gradient $m = (y_a - y_w)/(x_a - x_w)$

constant $c = (y_a - mx_a)$

Now, the problem arises on how to intersect this line with the spectrum locus. Unfortunately, the spectrum locus was produced by experimental data, Table IV eliminating any chance of finding any previously defined equation. Looking at the spectrum locus it is easy to see that no least squares method can be used to fit a curve to the data points. In fact it would be hard to use any curve fitting technique on the spectrum locus because of its curvature. Some form of spline fitting was considered but after some debate was dropped in favor of the following method.

The data points of Table IV were placed into a subroutine. This again removed the necessity to read them in via data cards. If there should come a time when these data points should be re-evaluated, as is

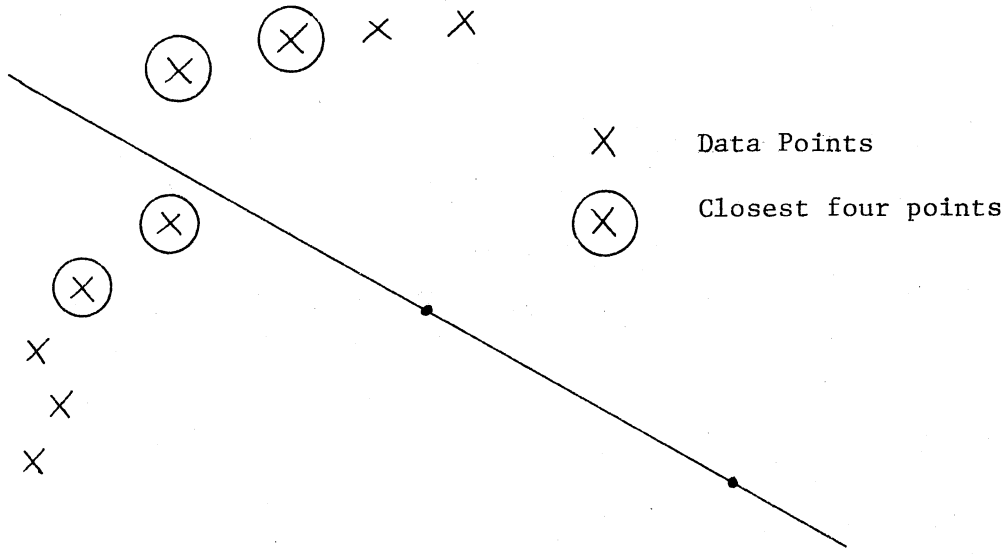


Figure 39. Straight line intersecting part of spectrum locus data points.

often the case with each advnace in spectrometric technology, they can easily be replaced. The subroutine then proceeds to find the closest four data points which bracket the equation of the line calculated earlier. Figure 39 illustrates this procedure.

To find the point of intersection, three methods were considered. The first was to pass a line through the closest two points and find the point of intersection between it and the previous line. Figure 40 illustrates this. The second method was to pass a quadratic equation through three of the closest points and intersect this with our straight line. Figure 41 illustrates this. It was thought that this would give a better result than the first method. The third method was to use the average of two quadratics. (Figure 39, 40, 41 are unrealistically bad cases.)

The first quadratic was to pass through the first three of the closest four points. The second quadratic was to pass through the last

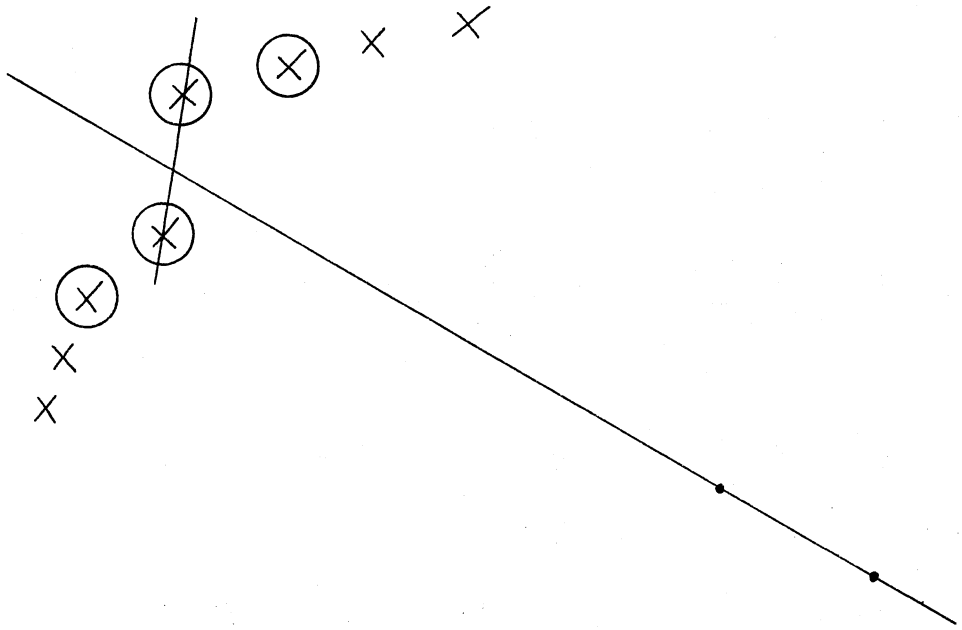


Figure 40. Intersection of two straight lines.

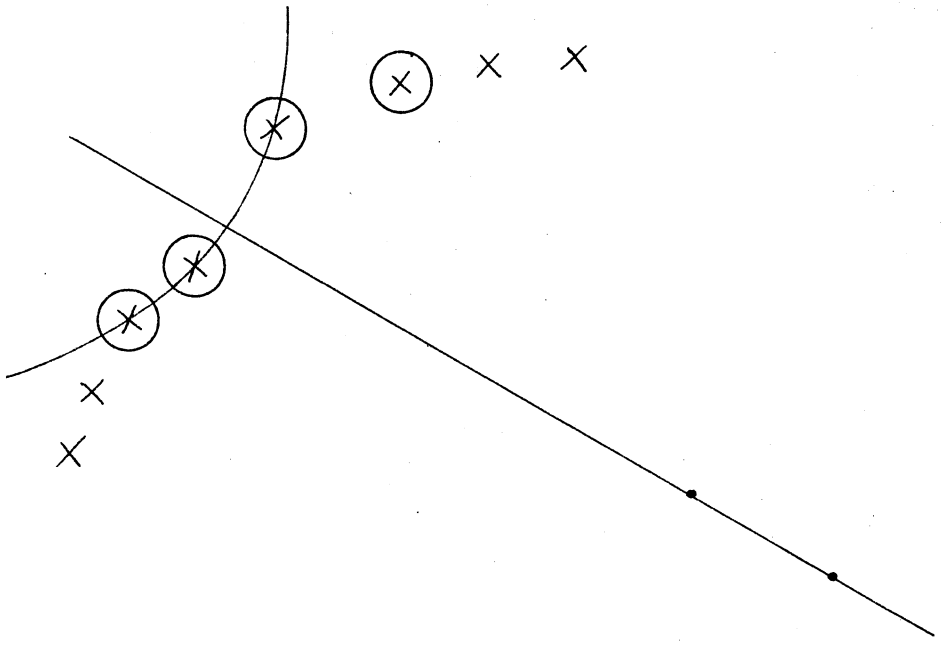


Figure 41. Intersection of a straight line and a quadratic.

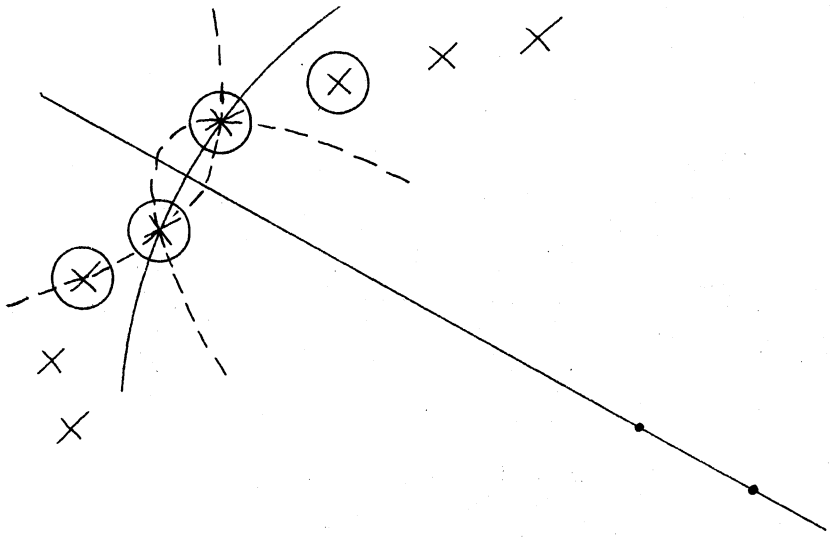


Figure 42. Intersection of a straight line and a quadratic which is the average of two quadratics.

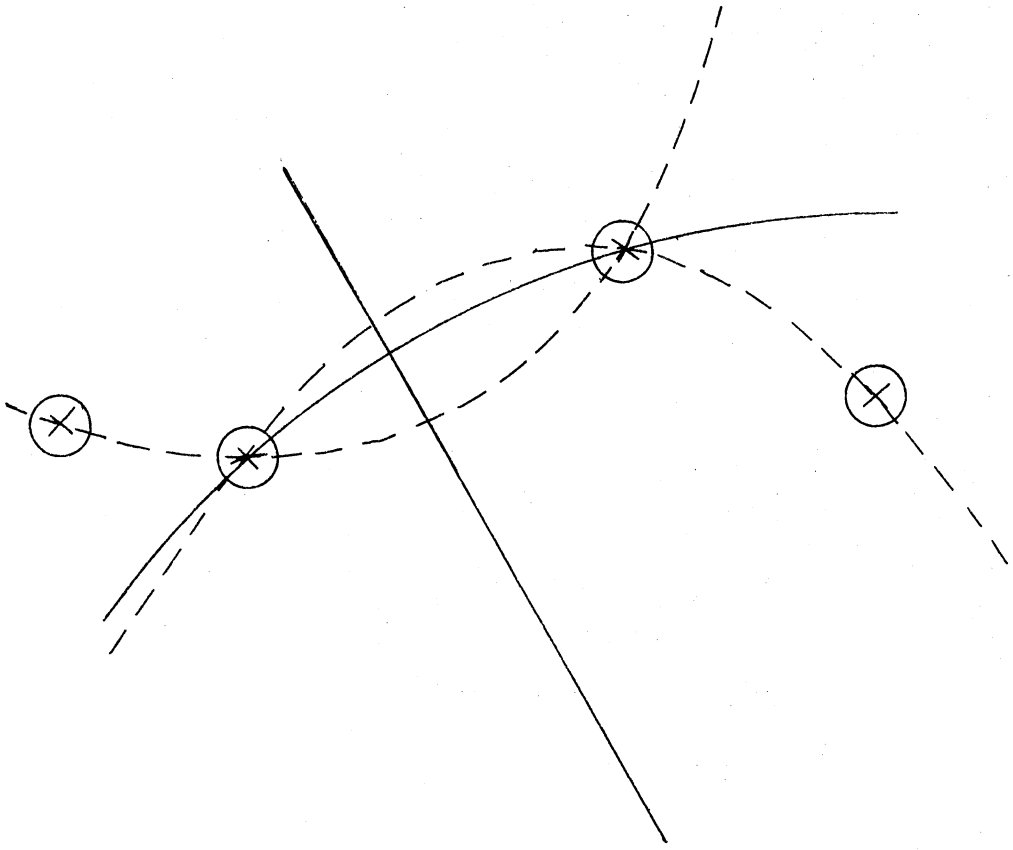


Figure 43. Rotation of quadratic points.

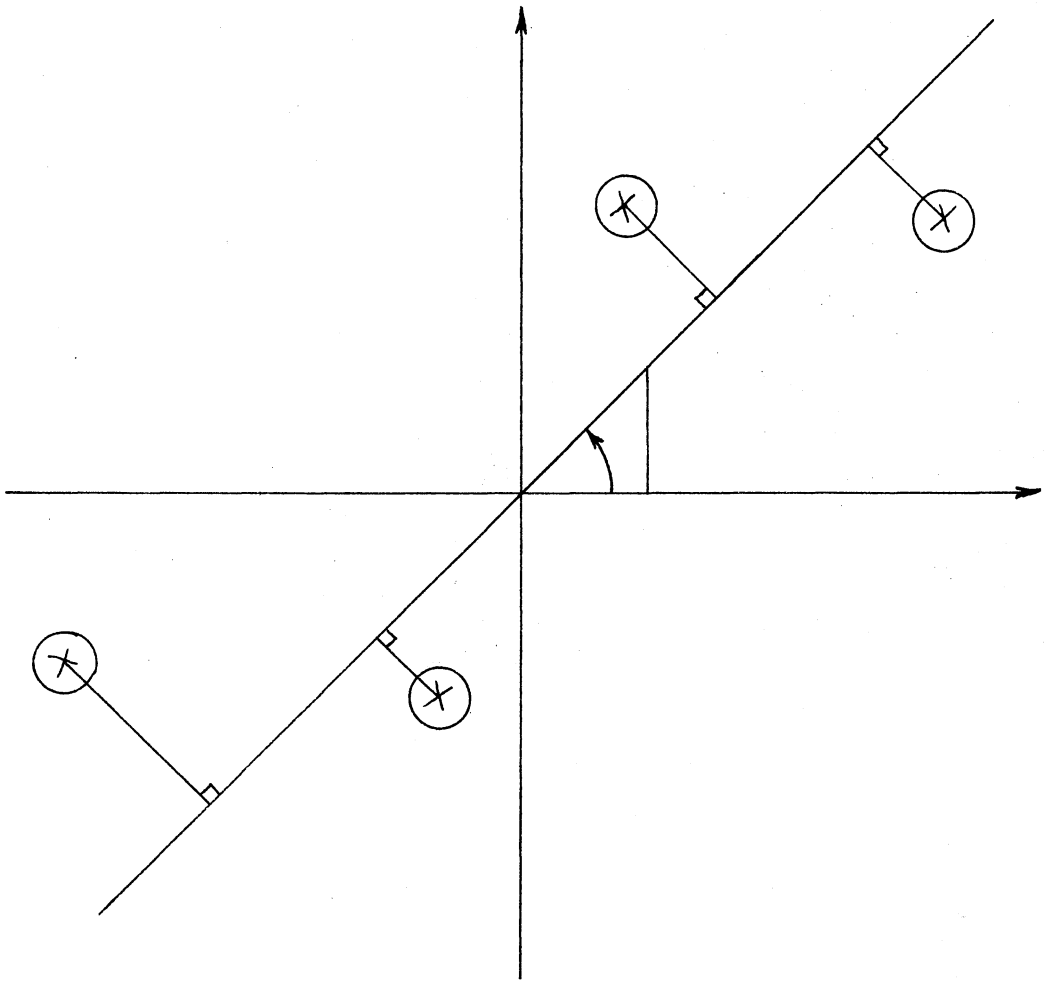


Figure 44. Straight line fit to a given four points.

three of the closest four points. The average of the two quadratics is constructed as follows:

Let $y = a_1x^2 + b_1x + c_1$ be the first quadratic.

$y = a_2x^2 + b_2x + c_2$ be the second quadratic.

The averaged quadratic is then

$$y = \frac{(a_1 + a_2)x^2}{2} + \frac{(b_1 + b_2)x}{2} + \frac{(c_1 + c_2)}{2} \tag{5.9}$$

Figure 42 illustrates this procedure. Again, it was thought that this method would produce better results than methods one and two. Method three was the one adopted.

Because of the curvature of the spectrum locus, some of the four point groups could produce problems if a quadratic was passed through them. Therefore, once the four points were found, it was decided to rotate them and the two points (x_a,y_a), (x,y) (forming the straight line) also, the constraint being that the four points after rotation should be in a configuration as nearly horizontal as possible. If this is obtained it would be easy to form the required quadratics. Figure 43 shows this.

To rotate the four points so that they were approximately horizontal, a straight line was fitted to the four points. Assuming error in both the x,y observations, using least squares a rotation formula was derived. Figure 44 shows this. The angle θ is equal to $TAN^{-1}(a)$. If the $TAN^{-1}(a)$ is found, then rotation of the four points to a horizontal position is trivial. The derivation to find $TAN^{-1}(a)$ goes as follows: Using the fact

$$(1,a)^T \cdot (-a,1) = 0$$

the projection matrix P(x,y) can be found.

Let $$P(x,y) = \begin{bmatrix} b & c \\ d & e \end{bmatrix}$$

Then $$\begin{bmatrix} b & c \\ d & e \end{bmatrix} \begin{bmatrix} -a \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} -ab + c = 0 \\ -ad + e = 0 \end{bmatrix}$$

and $$\begin{bmatrix} b & c \\ d & e \end{bmatrix} \begin{bmatrix} 1 \\ a \end{bmatrix} = \begin{bmatrix} 1 \\ a \end{bmatrix} \Rightarrow \begin{bmatrix} b + ac = 1 \\ d + ae = a \end{bmatrix}$$

Therefore $$c = ab \qquad b = 1 - ac$$

$$e = ad \qquad d = a - ae$$

So $$c = a(1-ac) = a - a^2c = (1+a^2)c = a \Rightarrow c = \frac{a}{1 + a^2}$$

$$c = ab \Rightarrow b = \frac{1}{1 + a^2}$$

and $$e = a(a - ae) = a^2 - a^2e$$

$$\Rightarrow (1 + a^2)e = a^2 \Rightarrow e = \frac{a^2}{1 + a^2}$$

$$e = ad \Rightarrow d = \frac{a}{1 + a^2}$$

$$P(x,y) = \frac{1}{a^2 + 1} \begin{bmatrix} 1 & a \\ a & a^2 \end{bmatrix} \tag{5.10}$$

check.

$$P \begin{pmatrix} -a \\ 1 \end{pmatrix} = \frac{1}{a^2 + 1} \begin{bmatrix} 1 & a \\ a & a^2 \end{bmatrix} \begin{pmatrix} -a \\ 1 \end{pmatrix} = \frac{1}{a^2 + 1} \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$P \begin{pmatrix} 1 \\ a \end{pmatrix} = \frac{1}{a^2 + 1} \begin{bmatrix} 1 & a \\ a & a^2 \end{bmatrix} \begin{pmatrix} 1 \\ a \end{pmatrix} = \frac{1}{a^2 + 1} \begin{bmatrix} a^2 + 1 \\ a + a^3 \end{bmatrix} = \begin{pmatrix} 1 \\ a \end{pmatrix}$$

data is (x_i, y_i) , $i = 1,n$

$$P \begin{pmatrix} x_i \\ y_i \end{pmatrix} = \frac{1}{a^2 + 1} \begin{pmatrix} 1 & a \\ a & a^2 \end{pmatrix} \begin{pmatrix} x_i \\ y_i \end{pmatrix} = \frac{1}{a^2 + 1} \begin{pmatrix} x_i + ay_i \\ ax_i + a^2 y_i \end{pmatrix} = \frac{x_i + ay_i}{1 + a^2} \begin{pmatrix} 1 \\ a \end{pmatrix}$$

$$P\begin{pmatrix} x_i \\ y_i \end{pmatrix} - \begin{pmatrix} x_i \\ y_i \end{pmatrix} = \begin{bmatrix} \dfrac{x_i + ay_i}{1 + a^2} - x_i \\ \dfrac{ax_i + a^2 y_i}{1 + a^2} - y_i \end{bmatrix}$$

So $$\left\| P\begin{pmatrix} x_i \\ y_i \end{pmatrix} - \begin{pmatrix} x_i \\ y_i \end{pmatrix} \right\| = \left[\frac{x_i + ay_i}{1 + a^2} - x_i\right]^2 + \left[\frac{ax_i + a^2 y_i}{1 + a^2} - y_i\right]^2$$

$$= \left[\frac{x_i + ay_i - x_i - a^2 x_i}{1 + a^2}\right]^2 + \left[\frac{ax_i + a^2 y_i - y_i - a^2 y_i}{1 + a^2}\right]^2$$

$$= \frac{1}{(1 + a^2)^2}\left[a(y_i - ax_i)\right]^2 + \frac{1}{(1 + a^2)^2}(ax_i - y_i)^2$$

$$= \frac{(ax_i - y_i)^2}{(1 + a^2)^2}(a^2 + 1)$$

$$= \frac{(ax_i - y_i)^2}{(a^2 + 1)}$$

Thus $$\sum_{i=1}^{n} \left\| P\begin{pmatrix} x_i \\ y_i \end{pmatrix} - \begin{pmatrix} x_i \\ y_i \end{pmatrix} \right\| = \frac{1}{a^2 + 1} \sum_{i=1}^{n} (ax_i - y_i)^2$$

$$\frac{\partial f}{\partial a} = \frac{(a^2 + 1) \sum_{i=1}^{n} 2(ax_i - y_i)x_i - (2a) \sum_{i=1}^{n} (ax_i - y_i)^2}{(a^2 + 1)^2} = 0$$

$$= \frac{\sum_{i=1}^{n} a^3 x_i^2 - \sum_{i=1}^{n} a^2 y_i x_i + \sum_{i=1}^{n} a x_i^2 - \sum_{i=1}^{n} y_i x_i - \sum_{i=1}^{n} a^3 x_i^2 + 2\sum_{i=1}^{n} a^2 x_i y_i - \sum_{i=1}^{n} a y_i^2}{(a^2 + 1)^2}$$

$$= \frac{a^2 \sum_{i=1}^{n} x_i y_i + a\sum_{i=1}^{n} (x^2 - y_i^2) - \sum_{i=1}^{n} x_i y_i}{(a^2 + 1)^2} = 0$$

Therefore $$a^2 \sum_{i=1}^{n} x_i y_i + a \sum_{i=1}^{n} (x_i^2 - y_i^2) - \sum_{i=1}^{n} x_i y_i = 0$$

where

$$a = \frac{-\sum_{1}^{n}(x_i^2 - y_i^2) \pm \sqrt{\left(\sum_{1}^{n}(x_i^2 - y_i^2)\right)^2 + 4\left(\sum_{i=1}^{n} x_i y_i\right)^2}}{2\sum_{i=1}^{n} x_i y_i} \tag{5.11}$$

Of course, equation (5.11) will produce two values for angle (a). The one chosen is the one which minimizes

$$P\begin{pmatrix} x_i \\ y_i \end{pmatrix} - \begin{pmatrix} x_i \\ y_i \end{pmatrix} \tag{5.12}$$

To rotate the plane through the angle (a) the following transformation was used.

$$x_R = x \cos(a) + y \sin(a) \tag{5.13}$$

$$y_R = y \cos(a) - x \sin(a) \tag{5.14}$$

where (x_R, y_R) is the rotated point.

Once the points have been rotated, the next task is to fit a quadratic to them. There are many methods and algorithms around which perform this task. It does not matter which method is used because however you find a quadratic which passes through 3 points, that quadratic is unique. An easy method, and the one which was used, is that of Lagrange.

Given 3 points (x_1,y_1), (x_2,y_2), (x_3,y_3)

Define

$$L_i = \prod_{\substack{j=1 \\ j\neq i}} \frac{x - x_i}{x_i - x_j}, \quad i = 1,2,3 \tag{5.15}$$

Then each L_i is a quadratic in x, and

$f(x) = \Sigma y_i L_i$ is the required quadratic.

Once both quadratics have been found, the averaged quadratic can be found as described earlier.

To find the intersection of the quadratic and the straight line is straightforward:

Let equation of straight line be

$$y = ax + b \tag{5.16}$$

Let equation of quadratic be

$$y = cx^2 + dx + e. \tag{5.17}$$

Now for intersection

$$ax + b = cx^2 + dx + e$$

Then,
$$cx^2 + (d - a)x + (e-d) = 0$$

and
$$x = \frac{-(d - a) \pm \sqrt{(d - a)^2 - 4c(e - d)}}{2c} \tag{5.18}$$

The root which minimizes (4.12) is chosen. Once x is known, y is known also.

To complete the method, this point of intersection has to be re-rotated to take it back to the original form. To accomplish this, the following transformation is used:

$$x_B = x_R \cos(a) - y_R \sin(a) \tag{5.19}$$

$$y_B = x_R \cos(a) + y_R \sin(a) \tag{5.20}$$

where (x_B,y_B) are now the required boundary points on the spectrum locus. The point (x_B,y_B) will be used later to calculate the chroma. It was not necessary to find the dominant wavelength. It was felt that this would be an added feature to the program. The dominant wavelength is calculated as follows.

Using the value t,

where
$$t = (y_a - y_w)/(x_a - x_w) \tag{5.21}$$

λ_d (or λ_c) can be found by table reference. If the value of t is greater than 1, then 1/t is used. This table was previously generated by Judd (9). It was decided to use this method because these are published tables and are recommended for use in most publications on color. The points used in the program are those listed under the column with

Illuminant C as the heading. If another illuminant is used, then the points in the program should be replaced by those of one of the other columns. The points for illuminant C were placed into the program. The dominant wavelength is then found from these points. If t lies between two values, then linear interpolation is used to find the dominant wavelength.

Knowing the dominant wavelength, unfortunately, does not imply that the hue is known. Reference to the value charts will show that the hue lines are nonlinear in nature. In order to find the hue for the sample point, it was decided to generate the hue lines and find the closest two which bracket the sample point. To accomplish this, the following method was used.

For each hue line contained on the 9 value charts, three point were generated with the following restrictions:

1) the coordinates of one point are the white point (0.3101, 0.3163)

2) the second point must be a value boundary point

3) the third point lies between 1,2

4) all three points must lie on the hue line.

Now, a unique quadratic may be generated which passes through all three points and corresponds to the hue line on the value chart. Figure 45 illustrates this. These points were taken from (9) and wired into the program.

Next the angles made by the white point and the 40 boundary points were computed using ATAN2. See Figure 46.

ATAN2 returns positive values for θ if θ lies in quadrants 1,2 and negative values if θ lies in quadrants 4,3 (see Figure 46.) It should be

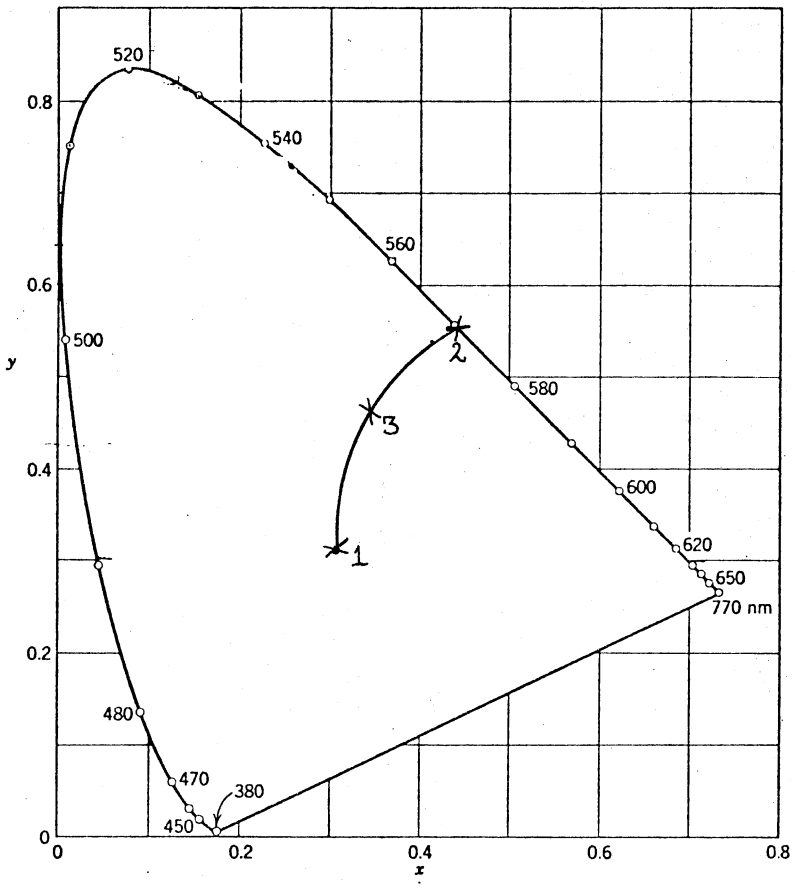


Figure 45. Unique quadratic passing through three hue points.

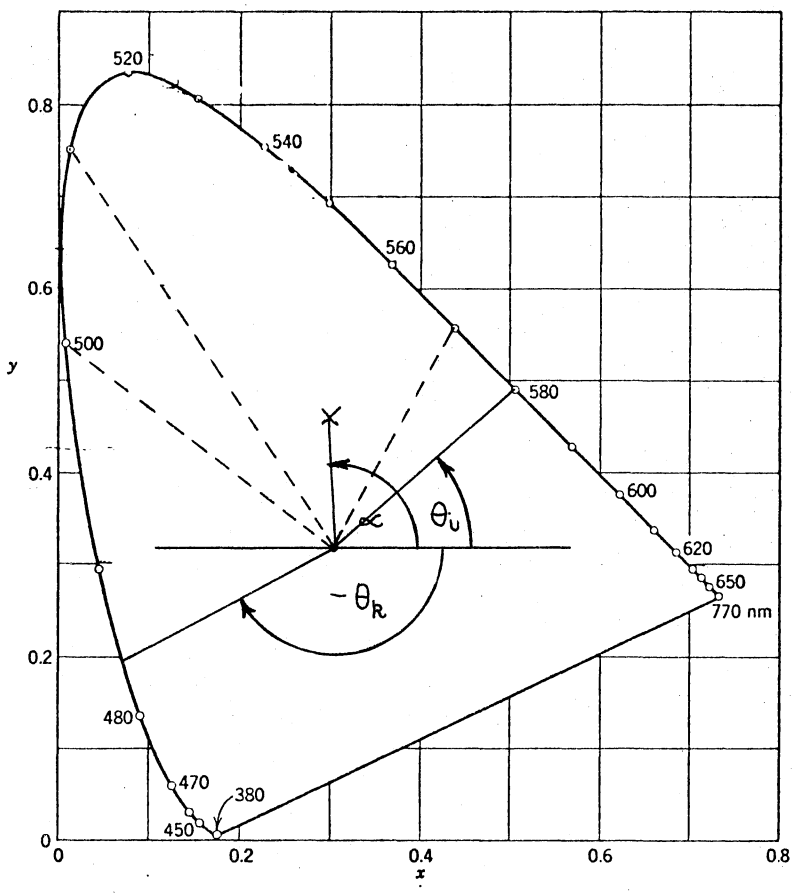


Figure 46. Angles calculated by ATAN2 function.

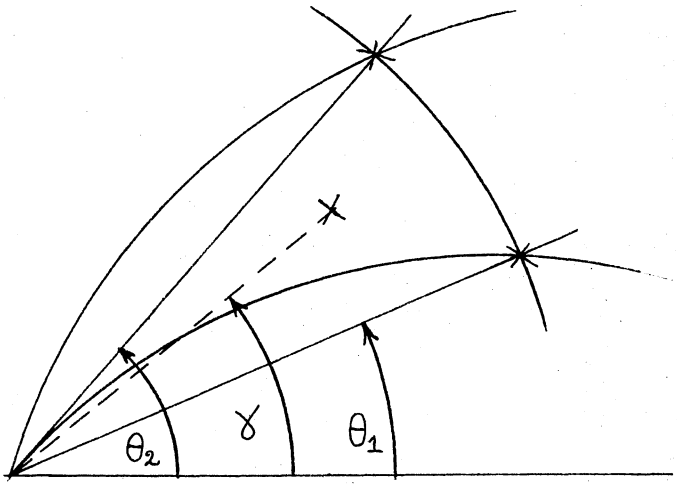


Figure 47. Method of bracketing sample point with hue lines.

noted that computing these angles makes the problem of finding the two hue lines, which bracket the sample point, independent of the x-y axis. Next the angle made by the sample points and the white point is computed, also using ATAN2. A linear search can now be used to find in between which two angles the sample angle falls. Unfortunately, a discontinuity appears at $+\Pi$ and $-\Pi$ and will give erroneous results in the search unless removed. If the angles in the search approach $+\Pi$ or $-\Pi$, the angles are rotated by a factor of 2Π and the search continued, thereby ensuring no discrepancies in the results. Once the two angles are found, then the hue can be calculated. See Figure 47.

Knowing the linear lines, the two nonlinear hue lines which bracket the sample lines can be generated. To do this, the points mentioned earlier on page 74 can now be used to generated two quadratics which will describe the two hue lines. Once generated, a check is made to see if the following two cases arise. See Figure 48 and 49.

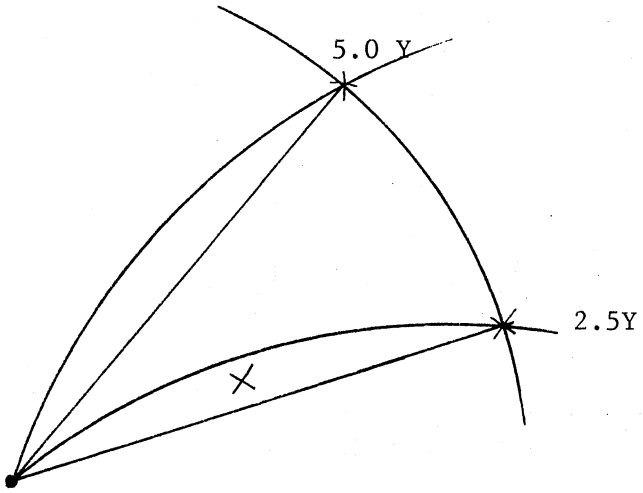


Figure 48. Case where sample point should move down one hue.

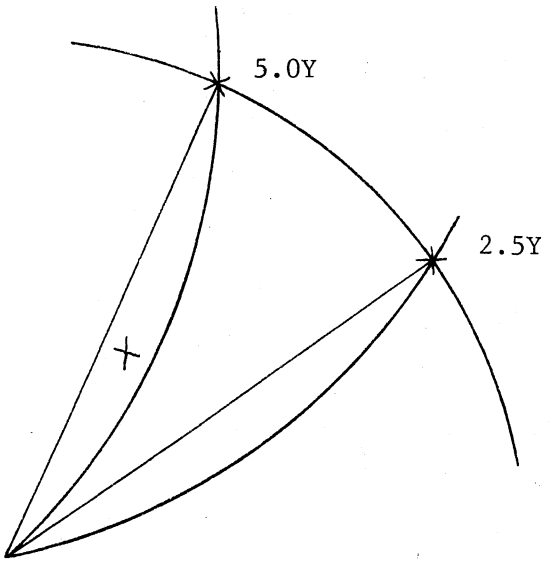


Figure 49. Case where sample point should move up one hue.

If either case occurs, the program takes corrective action. Once the two hues lines are generated, the hue for the sample point can now be determined. This is accomplished by first rotating the two quadratic hue curves and the sample point to a horizontal position by the method described earlier in the chapter. The hue is then determined by interpolating between the two hue curves. The hue and its dominant wavelength are now printed out. Knowing the hue, the chroma can be calculated. Chroma is found indirectly by use of excitation purity (P_e) and colorimetric purity (P_c). Before this is shown, a definition of each will be stated.

Excitation purity (P_e):

The ratio of two lengths on a chromaticity diagram, the first length being the distance between the point (x_w, y_w) and (x , y). The second length being the distance along the same direction from (x_w, y_w) to (x_B, y_B).

Quantity P_e is defined by the following relations.

$$P_e = \frac{(y - y_w)}{(y_B - y_w)} \quad \text{or} \quad P_e = \frac{(x - x_w)}{(x_B - x_w)} \tag{5.22}$$

Whether the formula in x or that in y is to be used depends on which gives the numerator the greater numerical value. For color stimuli for which no dominant wavelength exists the chromaticity coordinates to be taken for (x_B, y_B) are those of the corresponding point of the purple boundary.

Colorimetric purity (P_e):

Ratio of luminance of the spectrum light, in a mixture with a standard illuminant required to match the light considered, to the luminance of the mixture.

Quantity P_c is defined as follows:

$$P_c = (y_B/y)\cdot(P_e) \tag{5.23}$$

The physical principle stated by Munsell with respect to chroma was that in a sector-disc mixture test two equal areas of complementary hues should add to grey if they have the same VC product (V is value, C is chroma). As a result of this condition, the relation between colorimetric purity P_c, value and chroma for any hue must be:

$$P_c = (C/V)\cdot P_c(5/5)$$

Therefore

$$C = (VP_c)/P_c(5/5) \tag{5.24}$$

where

$$P_c = (y_B/y)\cdot(P_e)$$

The values for P_c (5/5) are listed in Table V.

Now knowing (x_w, y_w), $(x\ ,\ y\ )$, $(x_B,\ y_B)$, hue representation and P_c (5/5) the chroma can be calculated.

It should be noted that the results of the chroma and numeric hue calculations all hinge on factors which can vary. For example the endpoints for the hue representation are subjective choices. Should these change, the Munsell representation will change also. There is some disagreement also on Table V. If these should be reexamined and changed, the chroma will vary some also.

All the constants and tables that are used in the program were chosen from publications which stated that the tables given are used to complete Munsell color notation. The program was constructed so that if these tables should change, they could be replaced with the minimum amount of effort.

This chapter explained some of the concepts of color and the development of these concepts into a computer program. After the program was finished, data was obtained from Dr. Henrickson of the Oklahoma State Meat Lab and test run on the program. The test run results are contained in Chapter VI along with the conclusions of the thesis.

TABLE V

LIST OF Pc (5/5) VALUES

Hue	Pc
R 5/5	25.7
Y 5/5	64.2
G 5/5	28.1
B 5/5	20.7
P 5/5	0.8
BG 5/5	22.8
PB 5/5	9.1
RP 5/5	10.2
YR 5/5	56.9
GY 5/5	58.6

CHAPTER VI

RESULTS AND CONCLUSIONS

The data received from the Oklahoma State University Meat Laboratory were punched onto data cards and run through the computer program. The results are listed in Table VI. One column contains the results from the meat lab and the adjacent column contains the results from the computer program.

Before the conclusions, which are drawn from the listed results, are stated, the reader is reminded that the meat lab data are all contained within the RP-R-YR range of the spectrum. In fact, only in extreme cases do the data appear in either the RP or the YR range. The majority of the meat lab results lie in the R part of the spectrum, which is to be expected as the data relates to meat color.

Comparing the two results, it was found both agreed in all aspects except that of chroma. This can be attributed to two reasons. Firstly, the manual method used to interpolate between value charts and chroma loci could be in error or secondly the formula (5.24) used by the program could be wrong. To illustrate how this discrepancy can arise the follow example and statistical data are provided.

The following is an example of the manual method used to find the chroma of a given sample point.

Let $Y = 0.5217$, $x = 0.3313$, $y = 0.3351$, $v = 7.60$

Since $v = 7.60$ the chroma will be found by interpolation between

the charts for values 7 and 8 (Figures 32 and 33). On Figure 32 for x = 0.3313, y = 0.3351 the chroma is 1.3. On Figure 33 for x = 0.3313, y = 0.3351 the chroma is 1.4. Since 7.60 is .60 of the distance between value charts 7 and 8, the interpolated chroma will be

$$1.3 + [0.6(1.4 - 1.3)] = 1.36$$

The computer program does not use this form of interpolation. Instead, the equation (5.24) is used.

The chroma obtained from the manual results and the chroma produced by the program differ slightly. For the 20 sample points used, the following statistical data were calculated.

	Meat Lab Chroma	Program Chroma
Mean	5.793	6.6965
Variance	2.24527	2.6965
Standard deviation	1.49842	1.619032

It would appear that the program's estimation of the chroma is a little high. The spread of the data about the mean in both cases is about the same.

Reworking the Meat Lab data by hand it was found that there was a bias in the chroma results of 1.0. This was attributed to the interpolation method used on the small value charts. If this 1.0 is a true estimate of the variation, then this would make the chroma produced by the program reasonable.

It is felt that the chroma estimation is an area where further research could be applied. The mathematical concept of chroma is rather vague. The question of whether chroma can be computed by a mathematical means needs to be answered. This is a possible area of research. Another approach would be to place a table of chroma points into the program and use the same method as was employed in finding the hue.

Unfortunately this would produce a rather large table. It might be possible to reduce the table size by curve fitting techniques or some form of statistical analysis. A great deal of time would be required in this endeavor and could comprise another report or thesis.

In spite of the fact that there is a discrepancy in the chroma results it is felt that the purpose of this thesis was achieved and that a significant step has been made.

TABLE VI

LIST OF RESULTS PRODUCED BY THE COMPUTER PROGRAM

Calculation	Results	Computer Program Results	Meat Lab Results	Computer Program Results
Amber	31	31.0	32	32.0
Blue	17	17.0	16	16.0
Green	22	22.0	23	23.0
X	27.86	27.86	28.48	28.48
Y	22	22.00	23	23.00
Z	20.06	20.06	18.88	18.88
x	.399	.3985	.405	.4048
y	.315	.3146	.327	.3269
Hue	2.94R	2.86R	5.88R	5.73R
Value	5.24	5.24	5.34	5.34
Chroma	6.31	6.73	5.99	6.53

TABLE VI (Continued)

Calculation	Meat Lab Results	Computer Program Results	Meat Lab Results	Computer Program Results
Amber	31	31.0	30	30.0
Blue	17	17.0	17	17.0
Green	22	22.0	22	22.0
X	27.86	27.86	27.06	27.06
Y	22	22.00	22	22.00
Z	20.06	20.06	20.06	20.06
x	.399	0.3985	.392	.3915
y	.315	0.3146	.318	.3183
Hue	2.94R	2.86R	3.75R	3.64R
Value	5.24	5.24	5.24	5.24
Chroma	6.31	6.73	5.62	6.73

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APPENDIX

USER'S GUIDE

INPUT

The program uses a variable format. This allows the user to specify the input format. The variable format must be the first card in the data deck.

The second card is a title card. This allows the user to place a title on the output. The title may be 80 characters in length.

The data cards are placed behind the above two cards. The data is placed on the cards according to the format specified by the variable format card. It is very important that each number contained on the data card be integer and right justified. (Place the number in the rightmost position of its designated field.) Each set of filter data should be of the order amber, green, and blue; that is amber should be first, green second and blue last. (See Figure 51.)

To terminate the program a negative number must be placed in the amber field. (See Figure 52.) This card is placed at the end of the data cards. The program will continue to process data cards until it encounters the negative number in the amber field.

The program checks for blank data cards. If one is encountered a warning message and the position of the blank card in the data deck are printed. The blank card is rejected as valid data and the next data card is processed.

OUTPUT

Figure 50 illustrates a listing for output generated by the computer program using the data card in Figure 51.

lines(s)	
1	Title (user supplied.)
2	Number of data card being processed
3-13	Computer generated results
14-16	A warning message to inform the user that the program tends to over estimate when calculating the chroma
17-19	Formulas for calculating the variables listed in lines 3-13
20	The Munsell color notation
21	The dominant wavelength of the sample.

If any one of the amber, blue, and green fields are zero a warning message is printed to inform the user.

How to Execute the Program

Use the cards illustrated in figure 53. Do not change these cards or rearrange their order. If the control cards need to be updated, as is usually the case, then somebody in the Computing and Information Sciences should be contacted.

The program is set up to run on the WATFIV compiler but is designed to run on any system which supports the standard FORTRAN conventions.

JCL

Refer to Figure 53.

Card

1) Job card
   a) MUNSELL is the computer program name. This will appear on a banner page when the results are printed.
   b) NNNNN a five digit account number
   c) SS-SS-SSSS Social security number
   d) MEATLAB Control word
   e) CLASS=Z WATFIV auto batch

2) /*JOBPARM FORMS=101Z

This allows the output to be printed on the outside printer.

3) $JOB

a) TIME=9 specifies the amount of computer time in seconds the program is allowed to use.
b) PAGES=30 specifies the number of pages the program is allowed to print.
c) KP=029 Keypunch 029
d) LIST, CHECK, NOLIBLIST, WARN Control words.

4) CALL MSELL(5,6) invokes the Munsell color program.

a) 5 input unit number
b) 6 output unit number

5) END terminates the compiling of the program

6) $ENTRY control card

7) (3I3) variable format card

8) ***MEAT LAB DATA*** title card

9,10,11,12) data cards

13) -99 trailer card to terminate the program

14) $IBSYS Control card

15) // Control card

The output will be located by looking for a banner page with the word MUNSELL printed on it.

```
***MEAT LAB DATA***
RESULTS FOR DATA CARD   1
  AMBER       31.0
  BLUE        17.0
  GREEN       22.0
    X         27.86
    Y         22.00
    Z         20.06
   (X)         0.3985
   (Y)         0.3146
  HUE          2.86R
  VALUE        5.24
  CHROMA       6.73
  ***WARNING***
  THE CHROMA RESULT CALCULATED BY THE PROGRAM
  TENDS TO OVER ESTIMATE.

X = 0.80A + 0.18B          (X) = X / (X + Y + Z)
Y = 1.00G                  (Y) = Y / (X + Y + Z)
Z = 1.18B

MUNSELL COLOR NOTATION IS  2.86R   5.24 /  6.73
DOMINANT WAVELENGTH OF SAMPLE IS  619.58 NM
```

Figure 50. Computer Output Sample

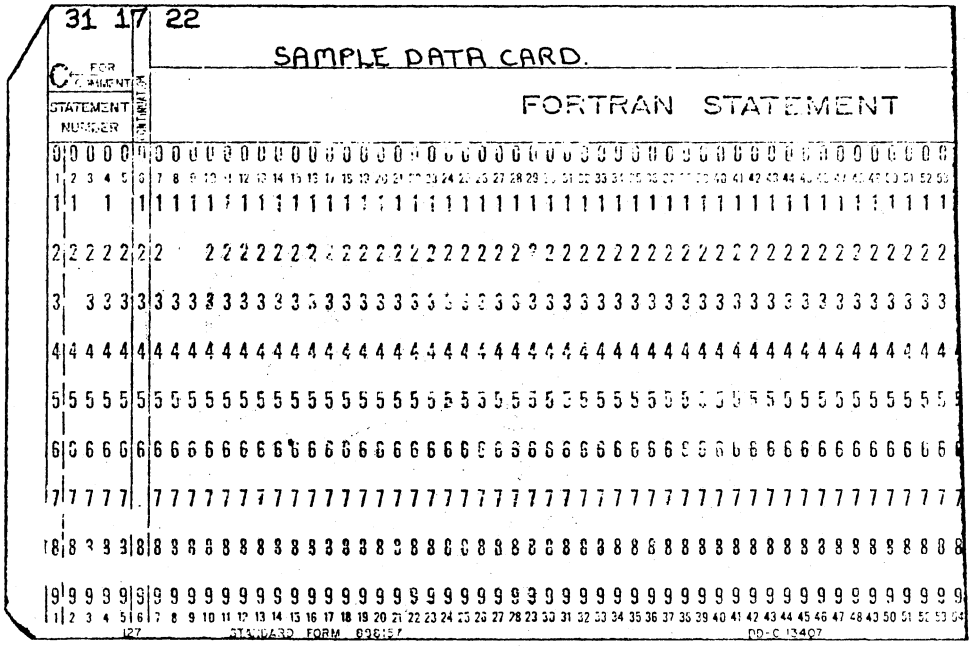

Figure 51. Sample data card.

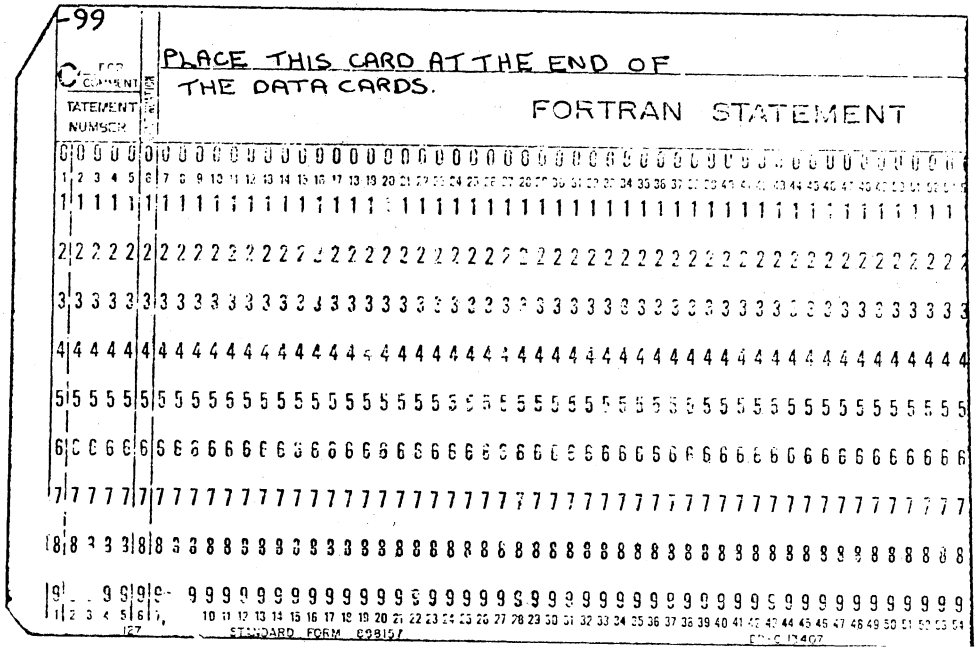

Figure 52. Trailer card for data deck.

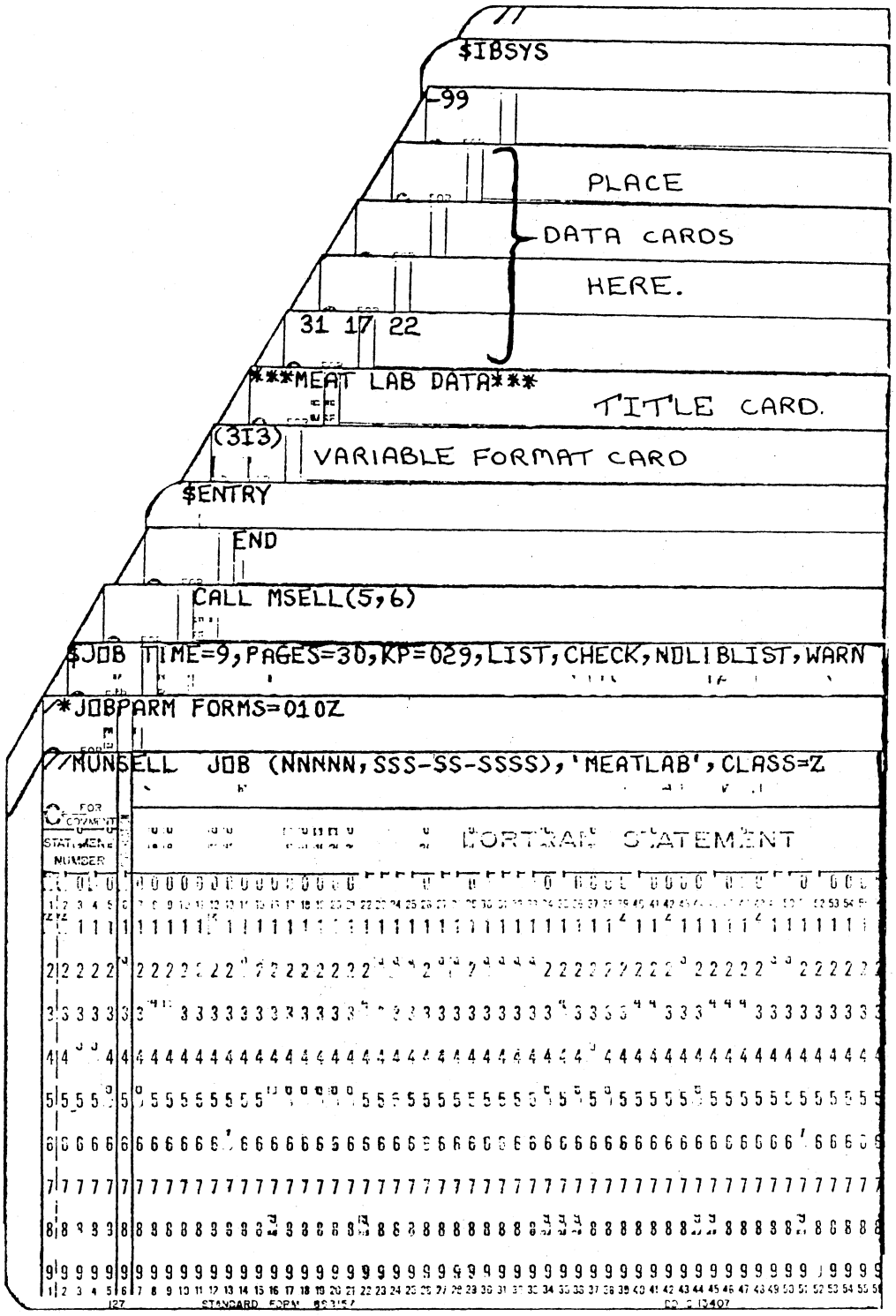

Figure 53. Sequence of cards to run color program.

VITA

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